```
> #TP 3 (MMI1)
> #Exo 1 : Babyloniens
> baby := proc(a,n)
  local i,u;
  u:=1;
  if (n=0) then return u;
  else
  for i from 1 to n do
  u:=1/2*(u+a/u);
  end do;
  end if;
  return u;
  end proc;
```

$$baby := \mathbf{proc}(a, n) \tag{1}$$

```
  local i, u;
  u := 1;
  if n = 0 then
      return u
  else
      for i to n do
          u := 1/2*u + 1/2*a/u
      end do
  end if;
  return u
end proc
```

```
> evalf(baby(2,2),30);
```

$$1.41666666666666666666666666667 \tag{2}$$

```
> evalf(baby(2,3),30);
```

$$1.41421568627450980392156862745 \tag{3}$$

```
> evalf(baby(2,4),30);
```

$$1.41421356237468991062629557889 \tag{4}$$

```
> evalf(baby(2,5),30);
```

$$1.41421356237309504880168962350 \tag{5}$$

```
> evalf(baby(2,10),30);
```

$$1.41421356237309504880168872421 \tag{6}$$

```
> evalf(sqrt(2),30);
```

$$1.41421356237309504880168872421 \tag{7}$$

```
> #Exo 2 : Tchebychev
```

```
> tcheb := proc(n)
  local i,A,B,C;
  A:=1;
  B:=x;
  if n=0 then return A;
  else if n=1 then return B;
  else
  for i from 2 to n do
  C:=2*x*B - A;
  A:=B;
  B:=C;
  end do;
  end if;
  end if;
  return C;
  end proc;
>
>
>
```

(8)

```
tcheb := proc(n)
    local i, A, B, C;
    A := 1;
    B := x;
    if n = 0 then
        return A
    else
        if n = 1 then
            return B
        else
            for i from 2 to n do
                C := 2 * x * B - A;
                A := B;
                B := C
            end do
        end if
    end if;
    return C
end proc
```

```
> tcheb(0), tcheb(1), tcheb(2), tcheb(10);
```

$$1, x, 2x^2-1, 2x\left(2x\left(2x\left(2x\left(2x\left(2x\left(2x^2-1\right)-x\right)-2x^2+1\right)-2x\left(2x^2-1\right)\right.\right.\right.\right.\right.\right. \quad (9)$$
$$\left.\left.\left.\left.\left.\left.+x\right)-2x\left(2x\left(2x^2-1\right)-x\right)+2x^2-1\right)-2x\left(2x\left(2x\left(2x^2-1\right)-x\right)-2x^2+1\right)\right.\right.\right.\right.\right.$$
$$\left.\left.\left.\left.+2x\left(2x^2-1\right)-x\right)-2x\left(2x\left(2x\left(2x\left(2x^2-1\right)-x\right)-2x^2+1\right)-2x\left(2x^2-1\right)+x\right)\right.\right.\right.\right.$$
$$\left.\left.\left.+2x\left(2x\left(2x^2-1\right)-x\right)-2x^2+1\right)-2x\left(2x\left(2x\left(2x\left(2x\left(2x^2-1\right)-x\right)-2x^2\right.\right.\right.\right.\right.$$
$$\left.\left.\left.\left.+1\right)-2x\left(2x^2-1\right)+x\right)-2x\left(2x\left(2x^2-1\right)-x\right)+2x^2-1\right)\right.\right.\right.$$
$$\left.\left.+2x\left(2x\left(2x\left(2x^2-1\right)-x\right)-2x^2+1\right)-2x\left(2x^2-1\right)\right.\right.$$
$$\left.\left.+x\right)-2x\left(2x\left(2x\left(2x\left(2x\left(2x\left(2x^2-1\right)-x\right)-2x^2+1\right)-2x\left(2x^2-1\right)\right.\right.\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left.\left.+x\right)-2x\left(2x\left(2x^2-1\right)-x\right)+2x^2-1\right)-2x\left(2x\left(2x\left(2x^2-1\right)-x\right)-2x^2+1\right)\right.\right.\right.\right.\right.$$
$$\left.\left.\left.\left.+2x\left(2x^2-1\right)-x\right)+2x\left(2x\left(2x\left(2x\left(2x^2-1\right)-x\right)-2x^2+1\right)-2x\left(2x^2-1\right)\right.\right.\right.\right.$$
$$\left.\left.\left.+x\right)-2x\left(2x\left(2x^2-1\right)-x\right)+2x^2-1\right.\right.$$

```
> sort(expand (tcheb(10)));
```

$$512x^{10}-1280x^8+1120x^6-400x^4+50x^2-1 \quad (10)$$

```
> eval(%,x=cos(a));
```

$$512\cos(a)^{10}-1280\cos(a)^8+1120\cos(a)^6-400\cos(a)^4+50\cos(a)^2-1 \quad (11)$$

```
> combine(%,trig);
```

$$\cos(10a) \quad (12)$$

```
> #Exo 3 :
> facto:=proc(n)
  local i,s;
  s:=1;
  if n=0 then return s; else
  for i from 1 to n do
  s:=s*i;
  end do;
  end if;
  return s;
  end proc;
>
```

$$facto := \mathbf{proc}(n) \quad (13)$$

```
local i, s;
s := 1;
if n = 0 then
    return s
else
    for i to n do
        s := s * i
```

```
        end do
    end if;
    return s
end proc
```

```
> facto(0);
```

$$1 \tag{14}$$

```
> facto(1);
```

$$1 \tag{15}$$

```
> facto(2);
```

$$2 \tag{16}$$

```
> facto(3);
```

$$6 \tag{17}$$

```
> facto(4);
```

$$24 \tag{18}$$

```
> facto(5),5!;
```

$$120, 120 \tag{19}$$

```
> factorec:=proc(n)
  if (n=0) then return 1;
  else return n*factorec(n-1);
  end if;
  end proc;
```

$$factorec := \mathbf{proc}(n)\ \mathbf{if}\ n = 0\ \mathbf{then}\ \mathbf{return}\ 1\ \mathbf{else}\ \mathbf{return}\ n * factorec(n - 1)\ \mathbf{end\ if}\ \mathbf{end\ proc} \tag{20}$$

```
> factorec(0);
```

$$1 \tag{21}$$

```
> factorec(1);
```

$$1 \tag{22}$$

```
> factorec(2);
```

$$2 \tag{23}$$

```
> factorec(3);
```

$$6 \tag{24}$$

```
> factorec(4);
```

$$24 \tag{25}$$

```
> factorec(5);
```

$$120 \tag{26}$$

```
> # Exo 4 :
> fibo:=proc(n)
  local i,a,b,c;
```

```
   a:=0;
   b:=1;
   if n=0 then return a; else if n=1 then return b; else
   for i from 2 to n do
   s:=a+b;
   a:=b;
   b:=s;
   end do;
   end if;
   end if;
   return s;
   end proc;
```

```
Warning, `s` is implicitly declared local to procedure `fibo`
```

(27)

```
fibo := proc(n)
    local i, a, b, c, s;
    a := 0;
    b := 1;
    if n = 0 then
        return a
    else
        if n = 1 then
            return b
        else
            for i from 2 to n do
                s := a + b;
                a := b;
                b := s
            end do
        end if
    end if;
    return s
end proc
```

```
> fibo(0);
```

0 (28)

```
> fibo(1);
```

1 (29)

```
> fibo(2);
```

(30)

```
                                    1                                (30)
> fibo(3);
                                    2                                (31)
> fibo(4);
                                    3                                (32)
> fibo(5);
                                    5                                (33)
> fibo(6);
                                    8                                (34)
> fibo(7);
                                   13                                (35)
> fibo(8);
                                   21                                (36)
> fibo(9);
                                   34                                (37)
> fibo(10);
                                   55                                (38)
> fiborec := proc(n)
  if n=0 then return 0; else if n=1 then return 1; else
  return fiborec(n-1)+fiborec(n-2);
  end if; end if;
  end proc;
fiborec := proc(n)                                                   (39)
    if n = 0 then
        return 0
    else
        if n = 1 then
            return 1
        else
            return fiborec(n - 1) + fiborec(n - 2)
        end if
    end if
end proc
> fiborec(0);
                                    0                                (40)
> fiborec(1);
                                    1                                (41)
```

```
> fiborec(2);
```

1 (42)

```
> fiborec(3);
```

2 (43)

```
> fiborec(4);
```

3 (44)

```
> fiborec(5);
```

5 (45)

```
> fiborec(6);
```

8 (46)

```
> fiborec(7);
```

13 (47)

```
> fiborec(8);
```

21 (48)

```
> fiborec(9);
```

34 (49)

```
> fiborec(10);
```

55 (50)

```
> #Exo 5 :
> restart;
> recurrente:=proc(f,u0,n)
  local i, u,t;
  t:=u0;
  if(n=0) then return t;
  else
  for i from 1 to n do
  u:=f(t);
  t:=u;
  end do;
  end if;
  return u;
  end proc;
```

recurrente := **proc** (*f*, *u0*, *n*) (51)

local *i*, *u*, *t*;

t := *u0*;

if *n* = 0 **then**

return *t*

else

```
        for i to n do
            u := f(t);
            t := u
        end do
    end if;
    return u
end proc
```

```
> recurrente2 := proc(f,u0,n)
  if (n=0) then return u0; else
  return f(recurrente2(f,u0,n-1));
  end if;
  end proc;
```

```
recurrente2 := proc(f, u0, n)
    if n = 0 then
        return u0
    else
        return f(recurrente2(f, u0, n - 1))
    end if
end proc
```
(52)

```
> f:=x->2*x;
```

$$f := x \to 2\,x \tag{53}$$

```
> recurrente(f,1,5),recurrente2(f,1,5);
```

$$32, 32 \tag{54}$$

```
> #Exerice 6 :
  feigenbaum := proc(a,u0,n)
  local i, u,t;
  t:=u0;
  if (n=0) then return t;
  else
  for i from 1 to n do
  u:=a*t*(1-t);
  t:=u;
  end do;
  end if;
  return u;
  end proc;
```

```
feigenbaum := proc(a, u0, n)
    local i, u, t;
    t := u0;
```
(55)

```
    if n = 0 then
        return t
    else
        for i to n do
            u := a * t * (1 − t);
            t := u
        end do
    end if;
    return u
end proc
```

```
> feigenbaum2 := proc(a,u0,n)
  if n=0 then return u0; else
  return a*feigenbaum2(a,u0,n-1)*(1-feigenbaum2(a,u0,n-1));end if;
  end proc;
```

```
feigenbaum2 := proc(a, u0, n)
    if n = 0 then
        return u0
    else
        return a * feigenbaum2(a, u0, n − 1) * (1 − feigenbaum2(a, u0, n − 1))
    end if
end proc
```

(56)

```
> feigenbaum(1.5,0.5,5), feigenbaum2(1.5,0.5,5);
```

0.3354052689, 0.3354052689 (57)

```
> feigenbaum(1.5,0.5,10), feigenbaum2(1.5,0.5,10);
```

0.3333973076, 0.3333973076 (58)

```
> feigenbaum(1.5,0.5,15), feigenbaum2(1.5,0.5,15);
```

0.3333353318, 0.3333353318 (59)

```
> #Exo 7 :
> addrec:=proc(m,n)
  if(n=0) then return m;
  else return addrec(m,n-1)+1;
  end if;
  end proc;
```

```
addrec := proc(m, n) if n = 0 then return m else return addrec(m, n − 1) + 1 end if end proc
```

(60)

```
> addrec(2,2);
```

4 (61)

```
> addrec(3,0);
```

(62)

```
                                  3                                  (62)
> addrec(3,5);
                                  8                                  (63)
> addrec(3,1);
                                  4                                  (64)
> multrec:=proc(m,n)
  if (n=0) then return 0;
  else return multrec(m,n-1)+m;
  end if;
  end proc;
```

$$multrec := \mathbf{proc}(m, n)\ \mathbf{if}\ n = 0\ \mathbf{then\ return}\ 0\ \mathbf{else\ return}\ multrec(m, n - 1) + m\ \mathbf{end\ if\ end\ proc} \quad (65)$$

```
> multrec(3,2);
                                  6                                  (66)
> multrec(3,0);
                                  0                                  (67)
> multrec(3,1);
                                  3                                  (68)
> multrec(3,5);
                                 15                                  (69)
> exprec:=proc(m,n)
  if (n=0) then return 1;
  else return exprec(m,n-1)*m;
  end if;
  end proc;
```

$$exprec := \mathbf{proc}(m, n)\ \mathbf{if}\ n = 0\ \mathbf{then\ return}\ 1\ \mathbf{else\ return}\ exprec(m, n - 1) * m\ \mathbf{end\ if\ end\ proc} \quad (70)$$

```
> exprec(0,0);
                                  1                                  (71)
> exprec(2,0);
                                  1                                  (72)
> exprec(3,1);
                                  3                                  (73)
> exprec(3,2);
                                  9                                  (74)
> exprec(3,3);
                                 27                                  (75)
> #Exo 8 :
> dichotomie:=proc(f,a,b,epsilon)
  local o,e,d,l;
```

```
   o:=a;
   e:=b;
   d:=evalf((a+b)/2);
   l:=evalf(abs(b-a));
   if epsilon > l then return d; else
   while (l>=epsilon) do
   if f(o)*f(d)>0 then o:=d;d:=evalf((o+e)/2);l:=evalf(abs(e-o));
   else e:=d;d:=evalf((o+e)/2);l:=evalf(abs(e-o));
   end if;
   end do;
   end if;
   return d;
   end proc;
```

$$dichotomie := \mathbf{proc}(f, a, b, \varepsilon) \tag{76}$$

```
local o, e, d, l;
o := a;
e := b;
d := evalf(1/2*a + 1/2*b);
l := evalf(abs(b - a));
if l < ε then
    return d
else
    while ε <= l do
        if 0 < f(o)*f(d) then
            o := d;
            d := evalf(1/2*o + 1/2*e);
            l := evalf(abs(e - o))
        else
            e := d;
            d := evalf(1/2*o + 1/2*e);
            l := evalf(abs(e - o))
        end if
    end do
end if;
return d
end proc
```

```
> dichotomie(sin,-Pi/2,Pi/2,0.0001);
```

$$-0.00004793689962 \tag{77}$$

```
> dichotomie(sin,-Pi/2,Pi/2,0.00000001);
```

$$-2.925836155\ 10^{-9} \tag{78}$$

```
> g:=x->x^5+3*x-7;
```

$$g := x \to x^5 + 3\,x - 7 \tag{79}$$

```
> dichotomie(g,0,2,0.001);
```

$$1.263183594 \tag{80}$$

```
> fsolve(g(x)=0);
```

$$1.262822860 \tag{81}$$

```
>
```