Speaker

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Title

Growing binary trees

Abstract

In this talk, we discuss a growing process that generates the family of binary trees. The process is as follows: we start with an anchor (that is, an active leaf), and at each step, we replace every anchor either by a (passive) leaf or by an internal node with two children that both are anchors themselves. Our interest is focused on active trees obtained using the above process (that is, on trees that possess at least one anchor).

Let us denote by $t_{n,m}$ the number of binary trees with n internal nodes and m anchors. When n is fixed and m varies, the total number of such trees is the Catalan number C_n. For a fixed value of m, we empirically observe that the proportion $t_{n,m}/C_n$ tends to a certain limit as $n \to \infty$, and provide some bounds. On the other hand, let us denote by a_n the number of nonzero elements $t_{n,m}$ (here, n is fixed and m varies). We establish the behavior of the sequence (a_n) and show that it is known in the literature as a meta-Fibonacci sequence. We also provide relations between the generating functions of growing binary trees and the so-called Mandelbrot polynomials, as well as an algorithm for generating a uniform growing binary tree of a given type.

Finally, we study the active binary trees with respect to height. Let $t_{n,m,h}$ be the number of active trees of height h with n internal nodes and m anchors. We establish the limit shapes of the nonzero domains $\{(n,k)\colon t_{n,2k,h} \neq 0\}$ as the height tends to infinity, as well as the limit shape of the nonzero domains $\{(k,h)\colon t_{n,2k,h} \neq 0\}$ as the number of internal nodes tends to infinity.

This talk is based on the ongoing work with Olivier Bodini and Antoine Genitrini.