On the Feynman graph expansion of 1-particle irreducible n-point functions in quantum field theory

Ângela Mestre

Institut de Minéralogie et de Physique des Milieux Condensés, Paris

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- Algebraic representation of graphs
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- Linear maps
- Graph generation and applications to QFT

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- antipode:

$$S(\phi(x_1) \dots \phi(x_n)) := (-1)^n \phi(x_1) \dots \phi(x_n)\,.$$

The ensemble of time-ordered *n*-point functions of a given type determine maps $\mathrm{S}(V) \to \mathbb{C}$ [Brouder *et al* 2004]:

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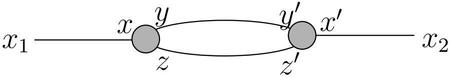

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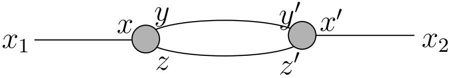

$$\int dxdydzdx'dy'dz'$$

$$G_F(x_1,x)\Sigma^{(3)}(x,y,z)G_F(y,y')G_F(z,z')\Sigma'^{(3)}(x',y',z')G_F(x',x_2)$$
$$= \int dydzdy'dz'\mathscr{V}^{(3)}(x_1,y,z)G_F^{-1}(y,y')G_F^{-1}(z,z')\mathscr{V}^{(3)}(x_2,y',z')\,.$$

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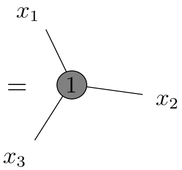
- $\phi(x_1)\phi(x_2)\phi(x_3) =$ 

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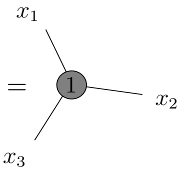
- $\phi(x_1)\phi(x_2)\phi(x_3) =$ 

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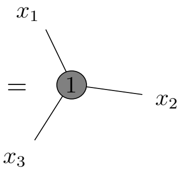
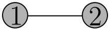
- $\phi(x_1)\phi(x_2)\phi(x_3) =$ 

- $R_{1,2} =$ 

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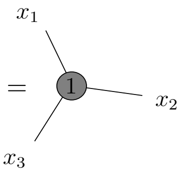
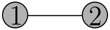
- $\phi(x_1)\phi(x_2)\phi(x_3) =$ 

- $R_{1,2} =$ 

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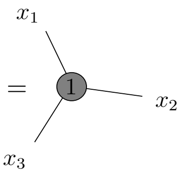
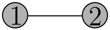
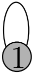
- $\phi(x_1)\phi(x_2)\phi(x_3) =$ 

- $R_{1,2} =$ 

- $R_{1,1} =$ 

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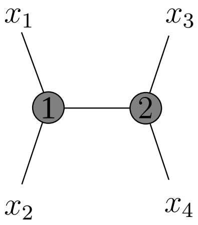

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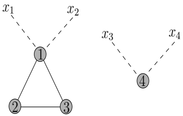
- $\gamma =$ 

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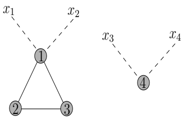
- $\gamma =$ 

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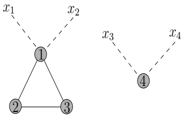
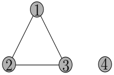
- $\gamma =$ 

- $\mathrm{int}^{\gamma}_{1,\ldots,4} =$ 

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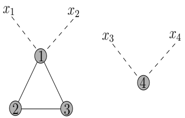
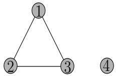
- $\gamma =$ 

- $\text{int}^{\gamma}_{1,\ldots,4} =$ 

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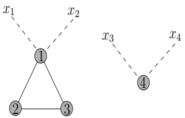
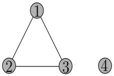
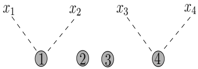
- $\gamma =$ 

- $\mathrm{int}^{\gamma}_{1,\dots,4} =$ 

- $\mathrm{ext}^{\gamma}_{1,\dots,4} =$ 

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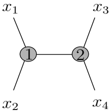
- $S_{1,2}^{\gamma} \in \mathsf{S}(V)^{\otimes 2} =$ 

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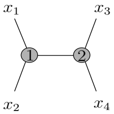
- $S^{\gamma}_{1,2} \in \mathsf{S}(V)^{\otimes 2} =$ 

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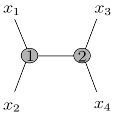
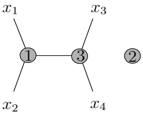
- $S^{\gamma}_{1,2} \in \mathsf{S}(V)^{\otimes 2} =$ 

- $S^{\gamma}_{1,3} \in \mathsf{S}(V)^{\otimes 3} =$ 

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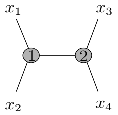
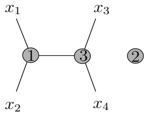
- $S^{\gamma}_{1,2} \in \mathrm{S}(V)^{\otimes 2} =$ 

- $S^{\gamma}_{1,3} \in \mathrm{S}(V)^{\otimes 3} =$ 

$(\ \sigma : \{1,2\} \to \{1,3\}; 1 \mapsto 1, 2 \mapsto 3)$

A non-associative non-commutative graph multiplication

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- $$\begin{aligned} &(u_1 \otimes \ldots \otimes u_v) \bullet_{i,j} (u'_1 \otimes \ldots \otimes u'_{v'}) := \\ &\quad (\tau_{v-1} \circ \ldots \circ \tau_i)(u_1 \otimes \ldots \otimes u_v) \otimes (\tau_2 \circ \ldots \circ \tau_j)(u'_1 \otimes \ldots \otimes u'_{v'}) \\ &\qquad = u_1 \otimes \ldots \otimes \hat{u}_i \otimes \ldots \otimes u_v \otimes u_i \otimes u'_j \otimes u'_1 \otimes \ldots \otimes \hat{u}'_j \otimes \ldots \otimes u'_{v'} \,. \end{aligned}$$

- $$S^{\gamma}_{1,\ldots,v} \bullet_{i,j} S^{\gamma'}_{1,\ldots,v'} := S^{\gamma}_{\sigma(1),\ldots,\sigma(v)} \cdot S^{\gamma'}_{\sigma'(1),\ldots,\sigma'(v')} \cdot$$

  $(S^{\gamma}_{\sigma(1),\ldots,\sigma(v)} \cdot S^{\gamma'}_{\sigma'(1),\ldots,\sigma'(v')} =$ disconnected graph$)$

Gluing two graphs at a vertex:

$$\lozenge_{i,j} := \tau_{v+v'-2} \circ \ldots \circ \tau_v \circ \cdot_v \circ \bullet_{i,j} : \mathsf{S}(V)^{\otimes v} \times \mathsf{S}(V)^{\otimes v'} \to \mathsf{S}(V)^{\otimes (v+v'-1)} .$$

$(v + v' - 1 =$ cut vertex$)$

Examples

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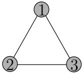
- $\gamma =$ 

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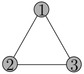
- $\gamma =$ 

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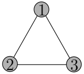
- $\gamma =$ 

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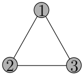
- $\gamma =$ 

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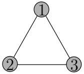
- $\gamma =$ 
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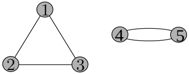
- $\gamma \bullet_{3,2} \gamma' =$ 

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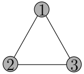
- $\gamma =$ 

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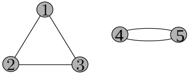
- $\gamma \bullet_{3,2} \gamma' =$ 

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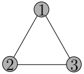
- $\gamma =$ 

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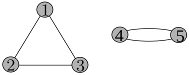
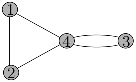
- $\gamma \bullet_{3,2} \gamma' =$ 

- $\gamma \Diamond_{3,2} \gamma' =$ 

A coalgebra structure on 1VI graphs

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- coproduct $\triangle : \mathbb{C}\mathfrak{B} \to \mathbb{C}\mathfrak{B} \otimes \mathbb{C}\mathfrak{B}$:

$$\begin{aligned} \triangle(\bar{s}) &:= \bar{s} \otimes \bar{s}\,; \\ \triangle(\bar{\gamma}) &:= \bar{s} \otimes \bar{\gamma} + \bar{\gamma} \otimes \bar{s} \quad \text{if} \quad \bar{\gamma} \neq \bar{s}\,; \end{aligned}$$

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$$\begin{aligned} \triangle(1) &:= 1 \otimes 1\,; \\ \triangle(B^{\gamma}_{1,\ldots,v}) &:= \frac{1}{v} \sum_{i=1}^{v} \triangle_i(B^{\gamma}_{1,\ldots,v})\,, \end{aligned}$$

where

$$\begin{aligned} \triangle_i(B^{\gamma}_{1,\ldots,v}) &:= B^{\gamma}_{\sigma_i(1),\ldots,\sigma_i(v)} + B^{\gamma}_{\sigma_{i+1}(1),\ldots,\sigma_{i+1}(v)} \\ &= B^{\gamma}_{1,\ldots,\widehat{i+1},i+2,\ldots,v+1} + B^{\gamma}_{1,\ldots,\hat{\imath},i+1\ldots,v+1}\,. \end{aligned}$$

- counit $\epsilon : \mathscr{B} \rightarrow \mathbb{C}$:

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The elements of $\mathscr{B}^*$ may be seen as monomials on tensors $B^{\gamma}_{\sigma(1),\ldots,\sigma(v)}$ with the componentwise product:

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Extend $\triangle := \frac{1}{v}\sum_{i=1}^{v} \triangle_i$ to $\mathscr{B}^*$ by requiring the maps $\triangle_i$ to satisfy:

$$\triangle_i(\prod_{a=1}^{k} B^{\gamma_a}_{\sigma_a(1),\ldots,\sigma_a(v_a)}) := \prod_{a=1}^{k} \triangle_i(B^{\gamma_a}_{\sigma_a(1),\ldots,\sigma_a(v_a)}) .$$

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Maps $Q^{(\rho)}_{i\geq 1}$ and $\hat{Q}^{(\rho)}_{i\geq 1}$

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Truncated coproduct: $\Delta_{\geq 1} : V^n \rightarrow \bigoplus_{i=1}^{n-1} V^i \otimes V^{n-i}$ [M. & Oeckl 2006]:

$$\Delta_{\geq 1}(1) = 0, \qquad \Delta_{\geq 1}(\phi(x)) = 0;$$
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$$Q_{i_{\geq 1}}^{(\rho)} := \frac{1}{2(\rho - 1)!} {R_{i,i+1}}^{\rho} \cdot \Delta_{i_{\geq 1}} : \mathsf{S}(V)^{\otimes v} \longrightarrow \mathsf{S}(V)^{\otimes v+1} \,.$$

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($Q_{i_{\geq 1}}^{(1)}$ [Glover *et al* 1979], [Livernet 2006])

Examples:

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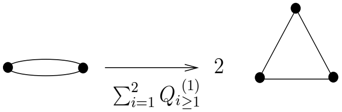
- 

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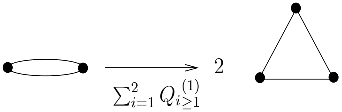
- 

On $\mathscr{B}^*$:

$$\hat{Q}_{i\geq 1}^{(\rho)}(\prod_{a=1}^{k} B^{\gamma_a}_{\sigma_a(1),\ldots,\sigma_a(v_a)}) := \frac{1}{2(\rho-1)!} {R_{i,i+1}}^{\rho} \prod_{a=1}^{k} \Delta_{i\geq 1}(B^{\gamma_a}_{\sigma_a(1),\ldots,\sigma_a(v_a)}) \,.$$

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- $$\begin{aligned} R_{3,4} \cdot \triangle_3(B^{C_3}_{1,2,3} \cdot B^{C_3}_{3,4,5}) &= R_{3,4} \cdot \triangle_3(B^{C_3}_{1,2,3}) \cdot \triangle_3(B^{C_3}_{3,4,5}) \\ &= R_{3,4} \cdot (B^{C_3}_{1,2,3} + B^{C_3}_{1,2,4}) \cdot (B^{C_3}_{3,5,6} + B^{C_3}_{4,5,6}) \\ &= R_{3,4} \cdot B^{C_3}_{1,2,3} \cdot B^{C_3}_{3,5,6} + R_{3,4} \cdot B^{C_3}_{1,2,3} \cdot B^{C_3}_{4,5,6} + \\ &\quad R_{3,4} \cdot B^{C_3}_{1,2,4} \cdot B^{C_3}_{3,5,6} + R_{3,4} \cdot B^{C_3}_{1,2,4} \cdot B^{C_3}_{4,5,6} \,; \end{aligned}$$

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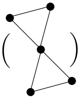
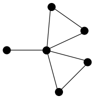
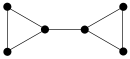
- $R_{3,4}\cdot\triangle_3$ () $= 2$  $+\ 2$ 

Generating 1VI Feynman graphs

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Pick out the terms that generate 1VI graphs according to a formula given in [M. 2009]:

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Theorem (see M. 2010)

For all integers $l \geq 0$ and $v > 1$, define $\mathfrak{V}^{l,v}_{1,\ldots,v} \in \mathrm{S}(V)^{\otimes v}$ by the following recursion relation:

- $\mathfrak{V}^{l,2}_{1,2} := \frac{1}{2(l+1)!} {R_{1,2}}^{l+1}$;
- $\mathfrak{V}^{0,v}_{1,\ldots,v} := 0$, $v > 2$;
-

$$\mathfrak{V}^{l,v}_{1,\ldots,v} := \frac{1}{l+v-1} \Bigg(\sum_{\rho=1}^{l+1} \sum_{i=1}^{v-1} Q^{(\rho)}_{i_{\geq 1}} (\mathfrak{V}^{l+1-\rho,v-1}_{1,\ldots,v-1}) + \sum_{j=2}^{v-2} \sum_{\rho=1}^{l-j+1} \hat{Q}^{(\rho)}_{v-1_{\geq 1}} (\mathfrak{B}^{l+1-\rho,v-1,j}_{1,\ldots,v-1}) \Bigg),$$

Theorem (cont.)

where for all integers $j > 1$, $v \geq j+1$ and $l \geq j$, $\mathfrak{B}^{l,v,j}_{1,\ldots,v}$ is given by the following recursion relation:

$$\mathfrak{B}^{l,v,2}_{1,\ldots,v} := \frac{1}{l+v-1} \sum_{l'=1}^{l-1} \sum_{v'=2}^{v-1} \sum_{i=1}^{v'} \sum_{j=1}^{v-v'+1} \left((l'+v'-1) \mathfrak{V}^{l',v'}_{1,\ldots,v'} \diamondsuit_{i,j} \mathfrak{V}^{l-l',v-v'+1}_{1,\ldots,v-v'+1} \right);$$

$$\mathfrak{B}^{l,v,j}_{1,\ldots,v} := \frac{1}{l+v-1} \sum_{l'=1}^{l-1} \sum_{v'=2}^{v-1} \sum_{i=1}^{v'-1} \left((l'+v'-1) \mathfrak{V}^{l',v'}_{1,\ldots,v'} \diamondsuit_{i,v-v'+1} \mathfrak{B}^{l-l',v-v'+1,j-1}_{1,\ldots,v-v'+1} \right).$$

Then, for fixed values of v and l, $\mathfrak{V}^{l,v}_{1,\ldots,v}$ is the weighted sum over all 1VI Feynman graphs with l loops, v vertices and no external edges nor self-loops, each with weight given by the inverse of its symmetry factor.

Generating 1PI Feynman graphs

Generating 1PI Feynman graphs

Theorem (see M. 2010)

For all integers $l > 0$ *and* $v > 1$, *define* $\mathfrak{I}^{v,l,}_{1,\dots,v} \in S(V)^{\otimes v}$ *by the following recursion relation:*

- $\mathfrak{I}^{l,2}_{1,2} := \mathfrak{V}^{l,2}_{1,2}$;
-

$$\mathfrak{I}^{l,v}_{1,\dots,v} := \mathfrak{V}^{l,v}_{1,\dots,v} + \frac{1}{l+v-1} \cdot \sum_{l'=1}^{l-1} \sum_{v'=2}^{v-1} \sum_{i=1}^{v-v'+1} \left((l'+v'-1) \mathfrak{V}^{l',v'}_{\pi_i(1),\dots,\pi_i(v)} \cdot \triangle_i^{v-1} \left(\mathfrak{I}^{v-v'+1,l-l'}_{1,\dots,v-v'+1} \right) \right), v > 2\,.$$

Then, for fixed values of v *and* l, $\mathfrak{I}^{l,v}_{1,\dots,v}$ *is the weighted sum over all 1PI Feynman graphs with* l *loops,* v *vertices and no external edges nor self-loops, each with weight given by the inverse of its symmetry factor.*

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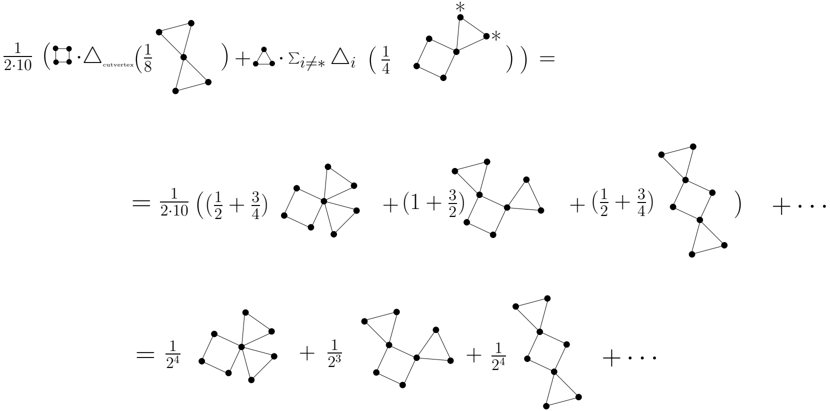

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Proposition

Fix an integer $n \geq 0$ as well as operator labels $x_1, \ldots, x_n$. For all integers $l \geq 1$, $l' \geq 0$ and $v \geq 1$, define $\Gamma^{l+l',v} : \mathrm{S}(V) \to \mathrm{S}(V)^{\otimes v}$ as follows:

$$\Gamma^{l',1} := \frac{1}{l'!} {T_1}^{l'} ;$$

$$\Gamma^{l+l',v} := \frac{1}{l'!} \mathfrak{I}^{l,v}_{1,\ldots,v} \cdot \delta^{v-1}({T_1}^{l'}) \cdot \Delta^{v-1}, v \geq 2 ,$$

Then, $\Gamma^{l+l',v}(\phi(x_1) \cdots \phi(x_n))$ is the weighted sum over all 1PI Feynman graphs with l loops, l' self-loops, v vertices and n external edges whose end points are labeled $x_1, \ldots, x_n$, each with weight given by the inverse of its symmetry factor.

Feynman graph expansion of 1PI n-point functions

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Contributions to the ensemble τ of 1PI *n*-point functions:

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Vertex order contributions:

Corollary

For $v \geq 1$:

$$\tau^{l+l',v} = \nu_{1PI}^{\otimes v} \circ \Gamma^{l+l',v}.$$

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