On a greedy algorithm for non-deterministic walks with several letters

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Abstract

A one-dimensional N-walk with several letters is a simplified version of a non-deterministic push-down automaton whose underlying digraph of transitions is a path. Each of the non-deterministic steps is a set of the possible stack operations. We study the acceptance probability of an empty word in the model where non-deterministic steps are chosen with given probabilities. We prove that the probability that the greedy algorithm finds an excursion compatible with a given N-path undergoes a phase transition.

Non-deterministic walks with several letters

- An N-walk is a sequence of sets of admissible steps.
- A step of Dyck type can be of the form \( x \) (adding a letter) or \( \overline{x} \) (removing a letter), where \( x \in \Sigma \).
- Furthermore, a step can be of the form \( \overline{xy} \): add a letter \( x \) if the stack head is \( y \).
- An excursion with several letters is a sequence of stack states, where each step either removes a top stack letter, or adds one.
- An N-excursion is an N-walk compatible with at least one deterministic excursion.

Proof idea:urrences and generating functions (GFs)

Let \( a_n, b_n, w_n^a, w_n^b, a_n^u, b_n^u \) denote the probabilities that the top letter of a greedy walk of size \( n \) is \( a \) or \( b \), and the weight of strictly positive excursions starting with the letters \( a \) or \( b \), and non-negative excursions starting with the letters \( a \) or \( b \) length \( n \). Let \( A(z), B(z), W_0(z), W_1(z), U_a(z) \) and \( U_b(z) \) be their GFs. Then,

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\begin{align*}
A(z) &= \frac{z}{1-z} \left( P(\{a\}) + P(\{\overline{a}\} \cap \overline{b}) + P(\{\overline{a}, a, \overline{b}, b\}) - z A(\overline{a}) \right) \\
B(z) &= \frac{z}{1-z} \left( P(\{b\}) + P(\{\overline{b}\} \cap \overline{a}) + P(\{\overline{b}, b, a, \overline{a}\}) - z B(\overline{a}) \right) \\
W_0(z) &= \frac{z}{1-z} \sum_{y \in \Sigma} U_a(z) (P(\{y\} | x) W_0(z) | y) \quad \text{for} \quad x \in \Sigma, \\
W_1(z) &= 1 + z U_a(z) \sum_{y \in \Sigma} U_a(z) (P(\{y\} | x) W_0(z) | y) \quad \text{for} \quad x \in \Sigma.
\end{align*}
\]

\( U_a(z) \) satisfy a system of quadratic equations. Next, \( W_0(\pm 1) = 1 - P(\{y\} | x) \) are valid formal solutions. If they correspond to principal branches of the generating functions, then the greedy algorithm succeeds with a positive probability. That is not always the case: another branch will become principal after coalescence of the multiple roots, which happens when \( \Delta = 0 \).