Asymptotic behavior of combinatorial structures

Proposal for an OeAD WTZ/PHC Amadeus Project
01/01/2023–31/12/2024

Contents

1 Scientific objectives

Combinatorial structures are omnipresent in nearly all scientific fields: for example, computer science uses graphs and maps as models for large networks and data structures, chemistry uses lattice paths as models for certain long polymers, and biology uses trees and networks to study the phylogeny in big complex systems. This list is by far not complete (and would easily exceed the page limits of a proposal), yet the three chosen examples exemplify a common theme: In many fields the behavior of *large structures*, or in other words, the *asymptotic behavior* is of key relevance. This is exactly the type of questions which are tackled and solved by the field of *Analytic Combinatorics*, to which this proposal belongs.

In recent years, many new techniques have been developed that have made it possible to solve long-standing open problems. Our general goal is to further develop these techniques and make the tools available to even more researchers. This project has the following goals:

1. proving universal phenomena involving a stretched exponential μ^{n^σ} in the asymptotic enumeration;
2. studying parameters of large random structures using catalytic variables;
3. deriving limit laws and efficient sampling methods (Boltzmann uniform random generation), thus providing methods and software code for a large scientific community.

Before one can start to ask questions on certain parameters (e.g., returns to the x-axis, average height) or their distribution, one needs to solve the basic counting problem. In (most and interesting) applications, no simple closed-form solution is available, however, the asymptotic counting problem is solvable.

2 Goals and methods

Our approach is based on generating functions $F(t) := \sum_{n\geq 0} f_n t^n$, where f_n is the number of structures of size n. Often, in order to determine f_n, one decomposes the structure by introducing one extra parameter, which is called "catalytic". For example, for walks of length n, a natural recursion makes use of the current final altitude k of the walk. Let $f_{n,k}$ be the corresponding counting sequence. Then we consider the bivariate generating function

$$F(t,u) := \sum_{n,k\geq 0} f_{n,k} t^n u^k, \qquad \text{such that} \qquad F(t,1) = F(t). \tag{1}$$

For thousands and thousands of combinatorial structures (see the books [16] and [11]), one gets a functional equation for $F(t,u)$, from which the challenge is to get the asymptotic behavior of f_n for large n. Generically, for constants $C, A, \mu, \sigma, \alpha, k \in \mathbb{R}$, it is[1]

$$f_n \sim C\, A^n\, \mu^{n^\sigma}\, n^\alpha\, \ln(n)^k. \tag{2}$$

This form is obtained, e.g., by analyzing the generating function in the complex plane: The nature of the dominant singularities (i.e. closest to the origin) gives the asymptotics of f_n.

Goal 1. Asymptotics involving stretched exponentials μ^{n^σ}

Several fundamental structures from computer science and combinatorics involve challenging asymptotics, for which often only crude bounds were known. The most common form of asymptotics is $f_n \sim C\, A^n\, n^\alpha$. However, for minimal automata [13], phylogenetic trees [10], and some Young tableaux [8], we recently proved that their asymptotics (2) involve a surprising occurrence of a stretched exponential μ^{n^σ}. Extending the approach of these works, we plan to prove that this so far very rarely encountered phenomenon is more common than expected.

Our method relies on a bijection with *decorated lattice paths* where the weight of the jumps are space and time dependent. This allows us to exploit the random walk 2D-drift to obtain the asymptotics with a subexponential precision. This generalizes our proof for compacted trees [14] and offers access to the analysis of many families of directed acyclic graphs.

Remarkably, in all these examples, the stretched exponential is of the form $\exp(c\, a_1 n^{1/3})$, where $c > 0$ depends naturally on the model and $a_1 \approx -2.338$ is the largest root of the Airy function $\mathrm{Ai}(z)$. The power $\sigma = 1/3$ seems to be universal, we plan to prove it by showing that $\mathrm{Ai}(z)$ is connected to the density of the limit law for the final altitude of the decorated paths.

In all examples we solved so far, the asymptotics are related to the Airy function, however, our method can handle more general recurrences a priori involving any σ and any D-finite function[2]. It remains unknown to exhibit a (natural) combinatorial structure with such unusual asymptotics; one idea could be to consider lattice paths with forbidden patterns (for which we developed a new approach in [1]).

[1] Sequences $(a_n)_{n\geq 0}$ and $(b_n)_{n\geq 0}$ are asymptotically equivalent ($a_n \sim b_n$), if and only if $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$.
[2] A D-finite function satisfies a linear differential equation with polynomial coefficients.

Goal 2. Asymptotics of functional equations

Catalytic equations have their origin in lattice path enumeration [3] and map enumeration [19]. They are typically of the shape

$$P(F(t,u),F_1(t),\dots,F_k(t),u,t)=0, \tag{3}$$

where all power series $F(t,u),F_1(t),\dots,F_k(t)$ are a priori unknowns, but fully determined by this equation. In [9], it is proven that these functions are then algebraic.

Recently, we proved (under natural positivity assumptions) that in the case $k=1$ there is a universal asymptotic behavior: the coefficient behave either like $C\,A^n\,n^{-3/2}$ or like $C\,A^n\,n^{-5/2}$. Furthermore, there is a universal central limit theorem for additional parameters [12]. It is expected that the same dichotomy holds for general positive catalytic equations.

Let us consider an example of such a catalytic equation, linked with maps and m-ary search trees. The efficient algorithmic properties of these trees are guaranteed by an operation called "rotation". Rotations from left to right define a *partial order* on trees of the same size, which is called the *Tamari lattice*. This lattice has deep connections with various objects in mathematics and computer science; it leads to the following catalytic equation for planar m-constellations (see [15, Theorem 4.1]):

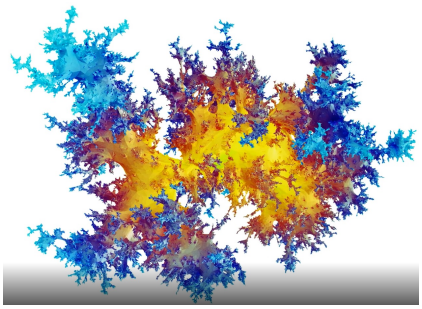
$$F(t,u)=1+ut(F(t,u)+\Delta)^{(m-1)}(F(t,u)). \tag{4}$$

Here, Δ is the divided difference operator: $\Delta S(u):=\frac{S(u)-S(1)}{u-1}$.

For many similar equations (or system of such equations), the challenge is to get the exponential growth A and the critical exponent α in the asymptotics (2). Following the methodology of our work [12], we expect to show that the critical exponent of a large class of such equations is $\alpha=-5/2$. If we now consider (non strongly-connected) systems of such equations, a larger variety of exponents occurs. However, following the methodology of our work [2] on context-free grammars, we expect to show that $\alpha=1/3$ never occurs.

Goal 3. Limit laws, limiting objects, random generation

For functional equations like the ones of Goal 2, and for equations of the type $F(t,u)=G(H(t,u))M(t)$, partial results exist for establishing the limit law of the parameter encoded by u. We will build on our works [1,3,6,11] for lattice paths, and [4,5,12,17,18] for composition schemes to establish conditions leading to Gaussian/Rayleigh/Airy limit laws, and other stable distributions. It includes, e.g., the study of connected components in different families of graphs or maps, for which the fluctuations are dictated by a distribution involving special functions like the Airy function. It is also solving some Pólya urn models [5, 7].



The picture on the left is a uniform sampling of our own of a very large map (see our Github package). Boltzmann methods give access to efficient random generation algorithms (see [16]), and to the limit laws of different parameters, which we plan to automatize with a package dedicated to lattice paths. This will allow us to tackle the question of local limit laws [18], and to analyze several statistics of the corresponding infinite objects.

3 Work plan and time schedule

The French team will have 5 sojourns of 5 days each (3 in 2023, and 2 in 2024): this corresponds to the typical budget mentioned on the PHC Amadeus French website.
The Austrian team will have 4 sojourns each year (5 days for the 2 professors, 10 days for each doctorant/postdoc): this corresponds to the budget mentioned on the Amadée project Austrian website. This thus leads to:

– Year 1: visits of Banderier/Fang/Singh to Austria, of Drmota/Stufler/Wallner/Hainzl to France.
– Year 2: visits of Haas/Banderier to Austria, of Drmota/Stufler/Hainzl/Wallner to France.

We expect a presentation of our results at the following yearly conferences: Aléa, FPSAC (Formal Power Series and Algebraic Combinatorics), and AofA (Analysis of Algorithms), which are the main events in our field. In 2023, these events will respectively be held in France at CIRM, at UC Davis, and at Taipei (2 of them have refereed published proceedings). Attendance at these events will be funded by our respective labs.

We also plan some applied computer science submissions in 2024 (such as STACS or Fun with Algorithms), with (open-source) packages of our own handling enumeration/limit laws/random generation of large classes of constrained combinatorial structures (mostly on links with lattice paths). Besides these conferences, as usual in our domains, we also plan to submit long journal versions of our works (in 2024).

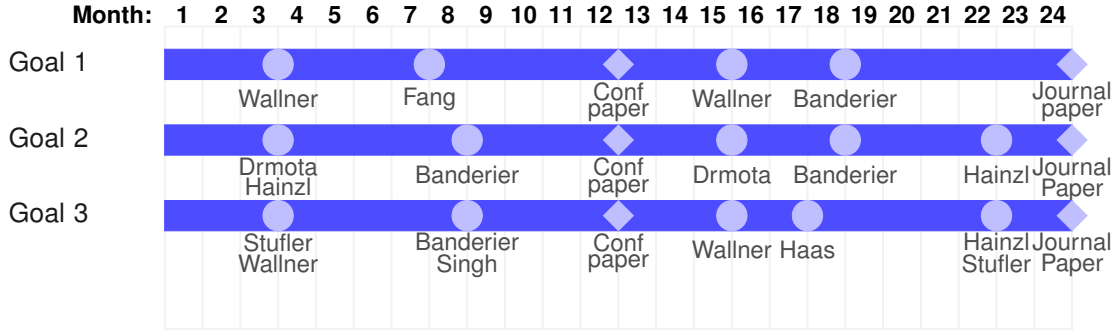


Figure 1: Prospective work with visits planned for 2023–2024, and the corresponding goals (1: stretched exponentials, 2: catalytic equations, 3: limit laws, as detailed in previous section).

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[5] Cyril Banderier, Markus Kuba, and Michael Wallner. Phase transitions of composition schemes: Mittag-Leffler and mixed Poisson distributions. *arXiv*, 2021.

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[13] Andrew Elvey Price, Wenjie Fang, and Michael Wallner. Asymptotics of minimal deterministic finite automata recognizing a finite binary language. In *AofA 2020*, LIPIcs, 2020.

[14] Andrew Elvey Price, Wenjie Fang, and Michael Wallner. Compacted binary trees admit a stretched exponential. *Journal of Combinatorial Theory, Series A*, 2021.

[15] Wenjie Fang. *Aspects énumératifs et bijectifs des cartes combinatoires : généralisation, unification et application*. PhD thesis, Université Paris Diderot, 2016.

[16] Philippe Flajolet and Robert Sedgewick. *Analytic Combinatorics*. Cambridge University Press, 2009.

[17] Bénédicte Haas, Christina Goldschmidt, and Delphin Sénizergues. Stable graphs: distributions and line-breaking construction. *To appear in Annales Henri Lebesgue*, 2022.

[18] Benedikt Stufler. Quenched local convergence of boltzmann planar maps. *Journal of Theoretical Probability*, 2021.

[19] William T. Tutte. A census of planar maps. *Canadian Journal of Mathematics*, 1963.

Further documents

The following sections correspond to the fields required by the online Austrian website. (The French fields are formulated slightly differently, but are also all covered by the next pages).

We submitted each of the next section as distinct independent .pdf on the Austrian website, as required. (For this reason, the next sections are not listed in the table of contents.) We glued them here altogether, for the referee convenience, to bring a more global look at once, and avoid to make him download several small PDFs, or ugly HTML forms.

Abstract

This proposal aims at shedding light on the large-scale behavior of fundamental combinatorial structures, such as trees, maps, random walks, ... Common questions concern universal properties: What are their asymptotics? Which limit laws are associated to parameters of such objects?

Our key tools are recurrences/generating functions, on which we apply (and develop) methods necessitating an interplay of analytic combinatorics and probability theory. This project then follows three interconnected research lines, which we summarized by the following goals.

First, we will consider several classes of objects (ranging from automata theory in computer science to phylogenetic trees in biology) for which the asymptotics remains a challenge. Via a powerful new method based on an interplay between non space-homogeneous random walks and differential equations, we will prove that they do involve a surprising stretched exponential $\mu^{n^{1/3}}$ in their asymptotic enumeration.

Second, we will study large random structures enumerated by an equation with a catalytic variable. Here, we shall prove a universal phenomenon: for some important classes of equations (covering enumeration of maps, lattice paths, ...), the asymptotics are universally of the form $n^{-5/2}$ or $n^{-3/2}$, thus bringing a rigorous proof for a heuristics from statistical mechanics.

Third, for several classes of functional equations, we will derive limit laws and efficient sampling methods (Boltzmann uniform random generation), thus providing methods and software code for a large scientific community.

Project partners

The researchers of the project have distinct positions: full researcher, full/associate university professor, postdoc, or PhD student. Such a scientific mix follows the philosophy of the PHC applications: allowing at the same time the project to reach its scientific goals, and allowing young researcher to connect with international research, paving the way to more thorough future collaborations them (postdoc positions in Austria/France for the 2 doctorants, ANR/ERC international projects for the other 2 younger participants).

The different participants are quite complementary, expert on different areas of combinatorics, while already sharing a common language (mostly via the use of generating functions), and a common interest (but with different expertise) on fundamental structures such as graphs, maps, trees, walks.

French participants:

Cyril Banderier (CNRS researcher, Univ. Paris Nord; age: 46) is a leading expert in analytic combinatorics. His work in this field was awarded a "Prix de l'Académie des Sciences" in 2013.

Wenjie Fang (maître de conf., Univ. Gustave Eiffel; age: 32) is a leading expert on enumerative and bijective combinatorics. In 2017, he is one of the 3 awardees for the best PhD thesis in computer science (Prix Gilles Kahn de la Société Informatique de France).

Bénédicte Haas (professeure, Univ. Paris Nord; age: 45) is a leading expert in probability theory (with a strong focus on trees and maps, fragmentation). She is involved in several scientific boards, e.g., she is editor-in-chief of the Electronic Journal of Probability.

Alexandros Singh (doctorant, Univ. Paris Nord, age: 27) started his PhD thesis in October 2019. He works on limiting behaviors (critical exponents) of maps and lambda-terms.

Austrian participants:

Michael Drmota (professor, TU Wien; age: 57) is a leading expert in analytic methods. He was president of the Austrian Mathematical Society (2010–2013), and he is corresponding member of the Austrian Academy of Science since 2013.

Eva-Maria Hainzl (doctorant, TU Wien; age: 32) started her PhD thesis in October 2020. She works on critical exponents of functions given by catalytic equations.

Benedikt Stufler (assistant professor, TU Wien; age: 35) is a young leading expert in combinatorial probability theory, also with a strong taste for programming. In 2016, he got awarded a research fellowship by the Deutsche Forschungsgemeinschaft.

Michael Wallner (postdoc, TU Wien; age: 34) is a young leading expert in analytic combinatorics. His PhD thesis got awarded a *Promotio sub auspiciis Praesidentis rei publicae* in 2017.

NB: All participants speak French, English and German.

Brief description of the partner institutions

TU Wien

The TU Wien is one of the main universities in Vienna, Austria. Under the mission statement "Technology for people", the university has received extensive international and domestic recognition in teaching and research. It has more than 28 000 students enrolled in 18 bachelor's, 33 master's, and 3 PhD programs; it has 8 faculties and about 5000 staff members (4000 scientists). The university's teaching and research focuses on computer science, quantum physics, engineering, and natural sciences. For more information see www.tuwien.at.

The Institute of Discrete Mathematics and Geometry – Combinatorics and Algorithms Group is the home institution of all researchers of the Austrian team. Its research focus lies in the analysis of algorithms as well as enumerative and analytic methods in combinatorics and number theory. Together with the combinatorics group of the University of Vienna, a joint seminar is held, which is attended by its many distinguished international guests.

Université Paris Nord

The University of Paris Nord was founded in 1968, after the split of the Université de Paris into 13 autonomous universities. It has more than 27 000 students spread in 28 bachelor and 92 master programs. It has $\sim$800 PhD students and $\sim$1000 academics (with a permanent position), most of them in its science faculty. For more information see www.univ-paris13.fr.

The French participants of the project (to the exception of one participant located in the Université Gustave-Eiffel) are spread between the computer science lab (in the team of combinatorics) and the mathematical lab (in the team of probability theory). Both of these teams have a very active seminar, and are extremely well connected to the international community, with regular participations to the main conferences of our domains. What is more, a "metalab" was recently created between the mathematical and the computer science lab (Fédération de Recherche CNRS "MathSTIC"), allowing professor invitations and postdoc sojourns, which we plan to use after this Amadeus project.

Tasks of the project participants

The researchers involved in the project have complementary expertise: The French team consists of three computer scientists and one mathematician (in probability theory). The members of the Austrian team are four mathematicians: One experienced researcher and his PhD student, one associate professor, one postdoc. Both teams work in fields related to analysis of algorithms and analytic combinatorics and there exist already several years of fruitful cooperations between several subgroups of the whole French-Austrian team such that they are able to communicate easily with each other.

The **Austrian team** has in-depth experience in methods of combinatorics, analysis, and probability theory:

- Michael Wallner is the project coordinator of the Austrian team. He is an early-stage researcher who has already acquired two large grants from the FWF (guaranteed until the end of this project in 2024), which provided him experience in administrating international projects. Together with Wenjie Fang from the French team and another colleague he has developed the method to prove stretched exponentials, which he will expand in Goal 1. Furthermore, he has in-depth experience in the use of generating functions to derive limit laws, which is why he will also contribute to Goal 3.
- Michael Drmota is a leading expert in analytic methods, specialized in the enumeration and limit laws of trees and maps. He will accordingly work on Goals 2 and 3.
- Eva-Maria Hainzl is doing her PhD under the supervision of Michael Drmota on catalytic variable equations, so Goal 2 will be her main area in the project.
- Benedikt Stufler will work on Goal 3, bringing all his insight from probability theory, and also his strong taste for programming (for the random sampling aspects).

The **French team** has broad experience in bijective combinatorics, analytic combinatorics, random generation, computer algebra, and probability theory:

- Cyril Banderier, French coordinator of the project, is involved in each of the Goals of this application.
- Wenjie Fang, expert in bijections and map enumeration, will work on Goal 1 (stretched exponential asymptotics) and Goal 2 (catalytic equation for maps).
- Bénédicte Haas, probability expert, will work the limit laws involved in Goal 3.
- Alexandros Singh is doing his PhD on the enumeration and limit laws of lambda-terms, a special class of directed acyclic graphs. He will contribute his expertise to Goal 3.

Further cooperation perspective

Our project contains a plan for research on which we are confident to get results within 2 years. Indeed, our problems are challenging and ambitious, but they have the merit to offer flexibility in the class of recurrences/functional equations that we will tackle, which is a good guarantee of getting new interesting results.

Natural generalizations of these recurrences (dealing, e.g., with more parameters) will be the next step, as recent progresses in multivariate complex analysis open the door to such so-far completely out-of-reach analysis. After the project will be finished, we thus plan to continue the cooperation within the framework of a larger bilateral project, a FWF-ANR Joint Project.

Moreover, the Austrian coordinator (Michael Wallner) has a running FWF Stand-alone project where it is planned to further cooperate with partners in Taiwan and Australia, whose expertise on non-conventional asymptotics will help to tackle other classes of equations. This would allow us to enlarge the project teams even further and would increase its scientific impact: we think of a large scale cooperation between Austria, France, Taiwan, and Australia (note that several of the French and Austrian participants to this project have already joint articles with researchers of these countries). Thus, we plan to extend this collaboration by participating (in 2024) to the annual project calls for such bilateral project from the Austrian, the French, and the Taiwanese national science foundations.

More locally, we consider jointly organizing a Special Research Program (SFB) with our Austrian colleagues as well as Innovative Training Networks (ITN) with our international colleagues, in order to found a European network building on the ALEA in Europe initiative. (This is the legacy of the late Philippe Flajolet, who managed in France to stimulate very fruitful interactions between researchers of probability theory, computer science, and statistical mechanics, which now meet yearly in the ALEA meeting at CIRM. It should be pinpointed that most of the recent advances on maps and random walk models are a direct consequence of the synergies created by this meeting.) Our project also aims, en passant, to create a dynamics for developing similar transdisciplinary interactions at a larger level in Austria and in France.

Last but not least, for the three young researchers (Benedikt Stufler, Wenjie Fang, and Michael Wallner), this project also furnishes the impetus and spectrum to help them to acquire an ERC starting grant and/or an FWF START Programme, to which they indeed plan to apply.

All these perspectives pinpoint the interdisciplinary success story of probability theory and combinatorics, offering en passant a wonderful occasion of wider formation for our younger researchers. All of this motivated this collaboration.

Cyril Banderier

Curriculum Vitæ

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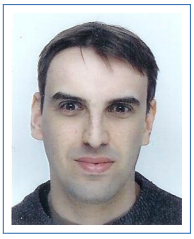
Citoyenneté: Français



Emploi

Univ. Sorbonne Paris Nord — **France**
CNRS researcher, Laboratoire d'Informatique de Paris Nord — *2002–...*

Max Planck Institut, Saarbrücken — **Allemagne**
Postdoc — *2001-2002*

Éducation

Univ. Paris 6 — **France**
Thèse: Combinatoire analytique des chemins et des cartes — *1998–2001*
(sous la direction de Philippe Flajolet, à l'INRIA)

Distinctions, invitations

Prix international de l'Académie des Sciences et de la Fondation Scientifique Franco-Taïwanaise — **2013**

Conférencier invité

- *Rencontres mathématiques in honor of Claude Dellacherie (Rouen, 2003),*
- *Colloquium for Philippe Flajolet's 60th Birthday (Paris, 2008),*
- *Philippe Flajolet and Analytic Combinatorics (Paris, 2011),*
- *École d'automne en analyse d'algorithmes et modèles aléatoires (Tunisie, 2012),*
- *Algorithmic and Enumerative Combinatorics (Autriche, 2013),*
- *CIMPA Summer School "Random structures, Analytic and Probabilistic Approaches" (Palestine, 2014),*
- *Conférence Lattice Paths (USA, 2015),*
- *Random Graphs (Cargèse, 2015),*
- *Symbolic Combinatorics and Differential Algebra (Canada, 2015)*
- *Analytic and Probabilistic Combinatorics (Canada, 2016),*
- *Analytic Workshop of the National Chiao Tung University (Taïwan, 2017),*
- *Asymptotic Algebraic Combinatorics (Canada, 2019),*
- *Prime numbers, determinism and pseudo-randomness in Memoriam of Christian Mauduit (CIRM, 2019),*
- *Conférence GASCOM (Italie, 2022).*

Vulgarisation scientifique

Conseil d'administration ou conseil scientifique de diverses associations de vulgarisation scientifique **2001–...**

MATh.en.JEANS, Science Ouverte, Maths pour Tous, etc.: coorganisation ou participation à des évènements annuels impliquant plus de 1000 visiteurs (congrès MATh.en.JEANS, Fête de la Science à la Cité des sciences et de l'industrie, Savante Banlieue), exposés grand public et tenue de stands.

Collaborations (in)ternationales

Projets binationaux

Porteur ou co-porteur de projets binationaux (PHC/ANR) avec : TU Wien (Autriche, 2005-2006, 2012-2013), Academia Sinica (Taïwan, 2013-2020), Université de Monastir (Tunisie, 2012), Université d'Antananarivo (Madagascar, 2006-2007), impliquant notamment des séjours de 1 semaine à plus d'un mois, des cours de master sur place, des exposés à des séminaires, etc.

Projets nationaux

Porteur du PEPS Holonomix (2012-1013), qui a notamment abouti à la création du GDR-EFI et à plusieurs projets ANR (dont notre ANR binationale avec Taïwan).

Sélection d'articles (since 2014)

1. C. Banderier, H.-K. Hwang, V. Ravelomanana, and V Zacharovas, *Analysis of an exhaustive search algorithm in random graphs and the n^{clogn}-asymptotics*. **SIAM J. Discrete Math**, 2014.
2. C. Banderier and M. Drmota, *Formulae and asymptotics for coefficients of algebraic functions*. **Combinatorics, Probability and Computing**, 2015.
3. C. Banderier and M. Wallner, *Lattice paths with catastrophes*. **Discrete Mathematics & Theoretical Computer Science**, 2017.
4. A. Asinowski, A. Bacher, C. Banderier, and B. Gittenberger, *Analytic combinatorics of lattice paths with forbidden patterns, the vectorial kernel method, and generating functions for pushdown automata*. **Algorithmica**, 2019.
5. C. Banderier, M. Wallner, *The kernel method for lattice paths below a line of rational slope*. **In Lattice Path Combinatorics and Applications. Developments in Mathematics Series (Springer)**, 2019.
6. C. Banderier and F. Luca, *On the period mod m of polynomially-recursive sequences: a case study*. **Journal of Integer Sequences**, 2019.
7. C. Banderier, P. Marchal, and M. Wallner, *Periodic Pólya urns, the density method, and asymptotics of Young tableaux*. **Annals of Probability**, 2020.
8. A. Asinowski, C. Banderier, and B. Hackl, *Flip-sort and combinatorial aspects of pop-stack sorting*. **Discrete Mathematics & Theoretical Computer Science**, 2021.
9. C. Banderier, F. Luca, F. Pappalardi, E. Treviño, and C. A. Gòmez Ruiz, *On Egyptian fractions of length 3*. **Revista de la Unión Matemática Argentina**, 2021.
10. C. Banderier, M. Kuba, M. Wallner, *Analytic combinatorics of composition schemes and phase transitions with mixed Poisson distributions*. **arXiv**, 2022.

Wenjie Fang

Curriculum Vitæ

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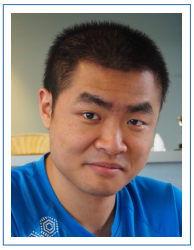

Employment

Université Gustave Eiffel — **Champs-sur-Marne**
Maître de conférences — *2019–now*

TU Graz
Postdoc, Institute of Discrete Mathematics — *2017–2019*

Thesis

Aspects énumératifs et bijectifs des cartes combinatoires — **2013–2016**
Supervised by Guillaume Chapuy and Mireille Bousquet-Mélou
Université Paris Diderot, defended at 11/10/2016, with distinction "*très honorable*"

Prizes

Prix de thèse Gilles Kahn - accessit — **2017**
Société Informatique de France

Selected publications

1. A. Elvey Price, W. Fang, and M. Wallner, *Compacted binary trees admit a stretched exponential*. **J. Combin. Theory Ser. A**, 2021.
2. W. Fang, H.-K. Hwang, and M. Kang, *Phase transitions from* $\exp(n^{1/2})$ *to* $\exp(n^{2/3})$ *in the asymptotics of banded plane partitions*. **J. Combin. Theory Ser. A**, 2021.
3. A. Elvey Price, W. Fang, and M. Wallner, *Asymptotics of minimal deterministic finite automata recognizing a finite binary language*. **Proceedings of AofA2020**, 2020.
4. O. Cooley, W. Fang, N.D. Giudice and M. Kang, *Subcritical random hypergraphs, high-order components and hypertrees*. **SIAM J. Discrete Math.**, 2020.
5. W. Fang and L.-F. Préville-Ratelle, *The enumeration of generalized Tamari intervals*. **European J. Combin.**, 2017.
6. G. Chapuy and W. Fang, *Generating functions of bipartite maps on orientable surfaces*. **Electron. J. Combin.**, 2016.

Bénédicte Haas

Curriculum Vitæ

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22/07/1976
Citizenship: French

Employment

Univ. Sorbonne Paris Nord — **France**
Full Professor, Laboratoire d'Analyse, Géométrie et Applications — *2015–current*

Université Paris-Dauphine — **France**
Assistant Professor — *2005–2015*

University of Oxford — **U.K.**
Postdoc, Department of Statistics — *2005–2015*

Education and diplomas

Univ. Paris-Dauphine — **France**
Habilitation, Title: Arbres aléatoires et fragmentations — *Nov. 2010*

Univ. Paris 6 — **France**
PhD in Probability Theory, Title: Fragmentations et perte de masse — *2001–2004*

Selected awards and distinctions

Prix des Annales de l'IHP — **Juin 2012**
For the paper "Behavior near the extinction time in self-similar fragmentations I: The stable case", written with Christina Goldschmidt

Selected editorial activity

Electronic Journal of Probability (EJP) — **2021–current**
Editor in Chief

Electronic Communications in Probability (ECP) — **2013–current**
Associate Editor

Selected publications

1. B. Haas and R. Stephenson, *Scaling limits of multi-type Markov branching trees*. **Probability Theory and Related Fields**, 2021.
2. N. Curien and B. Haas, *Random trees constructed by aggregation*. **Annales de l'Institut Fourier**, 2017.
3. N. Curien, B. Haas, and I. Kortchemski, *The CRT is the scaling limit of random dissections*. **Random Structures and Algorithms**, 2015.
4. C. Goldschmidt and B. Haas, *Behavior near the extinction time in self-similar fragmentations II: Finite dislocation measures*. **Annals of Probability**, 2014.
5. B. Haas and G. Miermont, *Self-similar scaling limits of Markov branching trees, with applications to Galton-Watson and random unordered trees*. **Annals of Probability**, 2012.

Alexandros Singh

Curriculum Vitæ

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Born 14/09/1994
Citizenship: Greek

Employment

Université Sorbonne Paris Nord — **France**
Teaching assistant (as PhD student) — *2021–now*

Education

Université Sorbonne Paris Nord — **France**
PhD student — *Oct. 2019–now*

National and Kapodistrian University of Athens — **Greece**
Master of Science in Algorithms, Logic, and Discrete Mathematics — *2017–2018*

Professional and academic experience

Member of the LambdaComb project (ANR PRC grant) — **2021–now**

Member of the organising committee for the 15th workshop Computational Logic and Applications CLA'20 — **2020**

Co-organiser of the graduate student seminar — **2019–now**
LIPN, Université Sorbonne Paris Nord

Selected publications

Journal articles:

1. A. Leivaditis, A. Singh, G. Stamoulis, D. M. Thilikos, K. Tsatsanis, and V. Velona, *Minor-obstructions for apex sub-unicyclic graphs*. **Discrete Applied Mathematics**, 2020.
2. A. Leivaditis, A. Singh, G. Stamoulis, D. M. Thilikos, and K. Tsatsanis, *Minor obstructions for apex-pseudoforests*. **Discrete Mathematics**, 2021.

Conference proceedings:

3. O. Bodini, A. Genitrini, M. Naima, and A. Singh, *Families of monotonic trees: Combinatorial enumeration and asymptotics*. **Computer Science – Theory and Applications**, 2020.
4. A. Leivaditis, A. Singh, G. Stamoulis, D. M. Thilikos, K. Tsatsanis, and V. Velona, *Minor-obstructions for apex sub-unicyclic graphs*. **EuroComb 2019, Acta Mathematica Universitatis Comenianae**, 2019.

Preprint:

5. O. Bodini, A. Singh, and N. Zeilberger, *Asymptotic distribution of parameters in trivalent maps and linear lambda terms*. **arXiv**, 2021.

Michael Drmota

Curriculum Vitæ

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Born 17/07/1964
Citizenship: Austrian

Employment

TU Wien **Austria**
Univ.Prof., Institut für Diskrete Mathematik und Geometrie *2006–current*

TU Wien **Austria**
Dean, Fakultät für Mathematik und Geoinformation *2013–2019*

TU Wien **Austria**
Assistant and Assoc.Prof., Institut für Algebra und Diskrete Mathematik *1986–2006*

Education

TU Wien **Austria**
Habilitation *1990*

TU Wien **Austria**
PhD (Dr. techn.) *1986*

Awards, national duties

Corresponding member of the Austrian Academy of Sciences **2013**

President of the Austrian Mathematical Society **2010–2013**

Promotio sub auspiciis Praesidentis rei publicae **1986**

Selected Publications

1. M. Drmota, C. Mauduit, and J. Rivat, *Prime numbers in two bases*. **Duke Mathematical Journal**, 2020.
2. M. Drmota, O. Gimenez, M. Noy, K. Panagiotou, and A. Steger, *The maximum degree of random planar graphs*. **Proceedings of the London Mathematical Society**, 2014.
3. M. Drmota, *Random Trees*. **Springer**, 2009.
4. M. Drmota, S. Janson, and R. Neininger, *A functional limit theorem for the profile of search trees*. **Annals of Applied Probability**, 2008.
5. M. Drmota, *An analytic approach to the height of binary search trees II*. **Journal of the ACM**, 2003.

Eva-Maria Hainzl

Curriculum Vitæ

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Born 05/11/1989
Citizenship: Austrian

Employment

TU Wien — **Vienna**
PhD Student (FWF Project) — *2020–now*

Karl-Franzens-University — **Graz**
Student assistant — *2019–2020*

University of Technology — **Graz**
Student assistant — *2017–2018*

Education

TU Wien — **Vienna**
PhD in Mathematics — *Since 2020*

University of Technology — **Graz**
Master (Dipl.Ing.) in Mathematics (with highest distinction) — *2019–2020*

Charles University — **Prague**
Erasmus stay — *2018–2019*

University of Technology — **Graz**
Bachelor (BSc.) in Mathematics (with highest distinction) — *2015–2018*

University of Music and Performing Arts — **Graz**
Magistra artium (Mag.art.) in Stage and Costume Design (with highest distinction) — *2010–2015*

Administrative duties

Member of the student board at Vienna School of Mathematics — **Since 2021**

Publications

1. M. Drmota and E.-M. Hainzl, *Universal properties of catalytic variable equations*. **To appear in AofA2022**, 2022.
2. O. Aichholzer, D. Eppstein, and E.-M. Hainzl, *Geometric dominating sets*. **To appear in a special issue devoted to EuroCG2021 of Computational Geometry: Theory and Applications**, 2022.
3. E.-M. Hainzl, D. Perz, J. Tkadlec, and M. Wallinger, *Finding a battleship of uncertain shape*. **Proceedings of EuroCG22**, 2022.
4. O. Aichholzer, D. Eppstein, and E.-M. Hainzl, *Geometric dominating sets - a minimum version of the no-three-in-line problem*. **Proceedings of EuroCG21**, 2021.

Benedikt Stufler

Curriculum Vitæ

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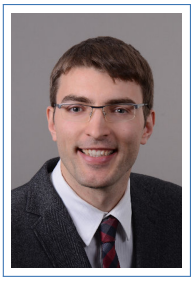

According to the OeAD guidelines, I am an *early-stage researcher* (PhD awarded 2015).

Employment

Vienna University of Technology — **Vienna**
Assistant Professor — *2020–now*

University of Munich — **Munich**
Postdoctoral Researcher — *2019–2020*

University of Zurich — **Zurich**
Postdoctoral Researcher — *2017–2019*

École Normale Supérieure de Lyon — **Lyon**
Postdoctoral Researcher — *2016–2017*

Education

Vienna University of Technology — **Vienna**
Habilitation (venia docendi) in Mathematics — *2022*

University of Munich — **Munich**
PhD (Dr. rer. nat.) in Mathematics — *2015*

Awards

Research Fellowship (STU 679/1-1) by DFG — **2016**

Selected Publications

1. B. Stufler, *Local convergence of random planar graphs*. **Journal of the European Mathematical Society**, 2021.
2. B. Stufler, *Graphon convergence of random cographs*. **Random Structures & Algorithms** 2021.
3. B. Stufler, *On the maximal offspring in a subcritical branching process*. **Electronic Journal of Probability**, 2020.
4. B. Stufler, *Local limits of large Galton–Watson trees rerooted at a random vertex*. **Annales de l'Institut Henri Poincaré - Probabilités et Statistiques**, 2019.
5. K. Panagiotou and B. Stufler, *Scaling limits of random Pólya trees*. **Probability Theory and Related Fields**, 2018.
6. K. Panagiotou, B. Stufler, and K. Weller, *Scaling limits of random graphs from subcritical classes*. **The Annals of Probability**, 2016.

Michael Wallner

Curriculum Vitæ

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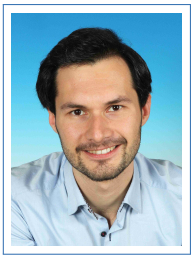
Citizenship: Austrian



According to the OeAD guidelines, I am an *early-stage researcher* (PhD awarded 2017).

Employment

TU Wien — **Vienna**
Postdoctoral Researcher — *2020–now*

Université de Bordeaux — **Bordeaux**
Postdoctoral Researcher — *2018–2020*

Academia Sinica — **Taipei**
Postdoctoral Researcher — *2017*

Education

TU Wien — **Vienna**
PhD (Dr. techn.) in Mathematics — *2017*

Grants

FWF Stand-Alone Project P 34142 — **Vienna**
Stretched exponentials and beyond, *TU Wien, PI, 400k€* — *2021–2024*

FWF Erwin Schrödinger-Fellowship J 4162 — **Bordeaux, Vienna**
Combinatorial and probabilistic study of higher dimensional lattice paths and tree-like structures, *Université de Bordeaux–TU Wien, PI, 157k€* — *2018–2021*

BMBWF Scholarship of excellence — **Bordeaux**
Exzellenzstipendium für sub auspiciis Praesidentis Prom., PI, 9k€ — *2017–2019*

Selected publications

1. A. Elvey Price, W. Fang, and M. Wallner, *Compacted binary trees admit a stretched exponential.* **Journal of Combinatorial Theory, Series A**, 2021.
2. A. Genitrini, B. Gittenberger, M. Kauers, and M. Wallner, *Asymptotic enumeration of compacted binary trees of bounded right height.* **Journal of Combinatorial Theory, Series A**, 2020.
3. C. Banderier, Ph. Marchal, and M. Wallner, *Periodic Pólya urns, the density method, and asymptotics of Young tableaux.* **Annals of Probability**, 2020.
4. M. Wallner, *A half-normal distribution scheme for generating functions.* **European Journal of Combinatorics**, 2020.
5. M. Wallner, *A bijection of plane increasing trees with relaxed binary trees of right height at most one.* **Theoretical Computer Science**, 2019.