Community Detection in Multiplex Networks using Locally Adaptive Random Walks

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July 25, 2015

Multiplex Networks

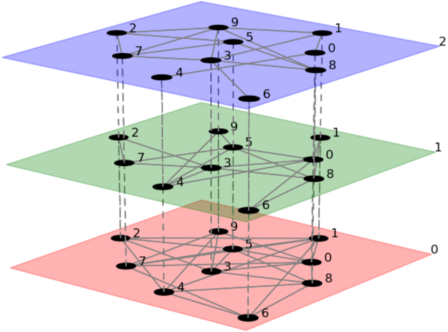

Figure: [Kivel, 2012]

Multiplex Networks

Definition: Multiplex Network

An L-layered multiplex network is a multi-layer undirected graph $\mathcal{M} = (V; A_k)_{k=1}^{L}$, where V is a set of nodes and A_k is the $N \times N$ adjacency matrix representing the set of edges in layer L_k for $k = 1, 2, ..., L$.

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- The connection between nodes v_i and v_j in L_k is given by $A_{ij;k} = A_{ji;k}$. Nodes v_i and v_j in L_k are neighbors if $A_{ij;k} = A_{ji;k} = 1$, otherwise $A_{ij;k} = 0$. Furthermore, $\forall k$, $A_{ij;k} = 0$ for $i = j$.

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- Each pair of corresponding nodes in different layers, v_i^k and v_i^l, has an inter-layer connection denoted by $\omega_{i;kl} \in \mathbb{R}$.

Multiplex Community Detection: Problem Formulation

Shared Communities

A shared community is a set of nodes for which several (but not necessarily all) layers provide topological evidence that these nodes form the same community that is shared across these layers.

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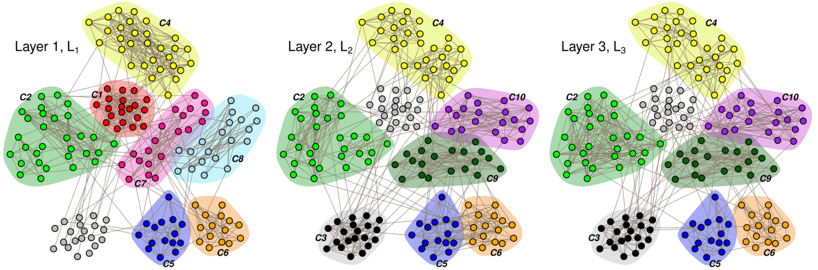

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4. Seed-centric algorithm extension [Hmimida and Kanawati, 2015].

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- Merge nodes using hierarchical clustering;
- Select best partition by maximizing the modularity function Q [Girvan and Newman, 2002].

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- Contribution: we adapt the transition probabilities of the random walk to depend on the local topological similarity between any pair of layers, at any given node.
- Result: the random walker spends longer times moving between nodes in communities which are shared across layers.
- Using properties of the random walk: introduce a dissimilarity measure between nodes and use it in a hierarchical clustering procedure to detect shared and non-shared communities.

LART: Inter-layer weights

Definition: Inter-layer weights

$$\omega_{i;kl} := |N_{i,k} \cap N_{i,l}|$$

where $N_{i,k} := \{v_j^k : A_{ij;k} = 1\}$ is the set of edges for v_i^k.

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$$\mathcal{A}^* := \left(\begin{array}{c|c|c|c} A_1 & W_{12} & \dots & W_{1L} \\ \hline W_{21} & A_2 & & \\ \hline \dots & & \dots & \\ \hline W_{L1} & & & A_L \end{array}\right).$$

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- We require $\mathcal{A}^*$ to be "well-behaved", i.e. connected and non-bipartite.
- Use $\mathcal{A}$ obtained from $\mathcal{A}^*$ by replacing the entry A_j with $A_j + \varepsilon I$ and W_{ij} with $W_{ij} + \varepsilon I$; here I is the $N \times N$ identity matrix and $0 < \varepsilon \leq 1$.

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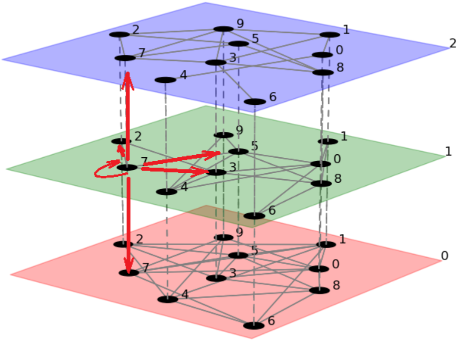

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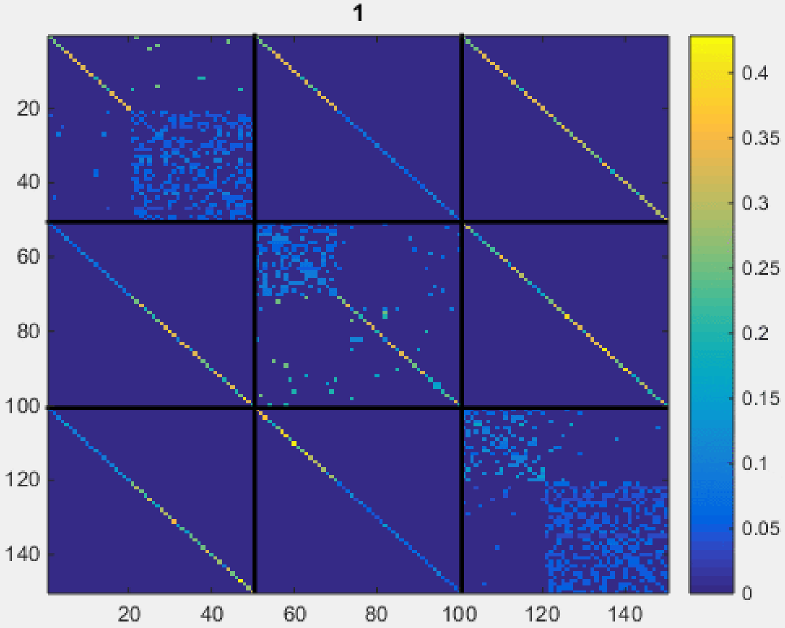
LART: Transition Probabilities

- The structure of $\mathcal{M}$ allows four possible moves that a random walker can make when in node v_i^k.
- The corresponding transition probabilities associated to these four possible moves are defined as

$$\mathcal{P}_{(i,k)(i,k)} := \frac{\varepsilon}{\kappa_{i,k}} \qquad \mathcal{P}_{(i,k)(j,k)} := \frac{A_{(i,k)(j,k)}}{\kappa_{i,k}}$$

$$\mathcal{P}_{(i,k)(i,l)} := \frac{\omega_{i;kl} + \varepsilon}{\kappa_{i,k}} \qquad \mathcal{P}_{(i,k)(j,l)} := 0$$

where $\kappa_{i,k}$ is the multiplex degree of node v_i^k in $\mathcal{A}$ defined as $\kappa_{i,k} := \sum_{j,l} \mathcal{A}_{(i,k)(j,l)}$.


1
20
40
60
80
100
120
140
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

LART: Node Dissimilarity Measure

Node Dissimilarity Measure - Same Layer

When v_i^k and v_j^k are in the same layer, their dissimilarity is defined as:

$$S(t)_{(i,k)(j,k)} := \sqrt{\sum_{h=1}^{N}\sum_{m=1}^{L} \frac{\left(\mathcal{P}^t_{(i,k)(h,m)} - \mathcal{P}^t_{(j,k)(h,m)}\right)^2}{\kappa_{(h,m)}}}.$$

Node Dissimilarity Measure - Different Layers

When v_i^k and v_j^l are in two different layers, L_k and L_l, we define the dissimilarity as:

$$S(t)_{(i,k)(j,l)} := \sqrt{s_1 + s_2 + s_3}$$

where

$$s_1 := \sum_{h=1}^{N} \left(\frac{\mathcal{P}^t_{(i,k)(h,k)}}{\sqrt{\kappa_{(h,k)}}} - \frac{\mathcal{P}^t_{(j,l)(h,l)}}{\sqrt{\kappa_{(h,l)}}} \right)^2$$

$$s_2 := \sum_{h=1}^{N} \left(\frac{\mathcal{P}^t_{(i,k)(h,l)}}{\sqrt{\kappa_{(h,l)}}} - \frac{\mathcal{P}^t_{(j,l)(h,k)}}{\sqrt{\kappa_{(h,k)}}} \right)^2$$

$$s_3 := \sum_{h=1}^{N} \sum_{\substack{m=1;\\ m \neq k,l}}^{L} \frac{\left(\mathcal{P}^t_{(i,k)(h,m)} - \mathcal{P}^t_{(j,l)(h,m)}\right)^2}{\kappa_{(h,m)}}.$$

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- We use the multiplex modularity Q_M proposed in [Mucha et al., 2010] as a criterion to select the best partition:

$$Q_M(\gamma) = \frac{1}{2\mu} \sum_{C \in \pi} \left[\sum_{(i,k),(i,l) \in C} \omega_{i;kl} + \sum_{(i,k),(i,l) \in C} [A_{ij;k} - \gamma_k \frac{d_{i,k} d_{j,k}}{2\mu_k}] \right]$$

where $2\mu = \sum_{i,j,k} A_{i,j;k}$, $d_{i,k} = \sum_j A_{ij;k}$, π is the partition into communities C, and γ_k is the resolution parameter for layer L_k.

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  - Mixture of different community structures (S5) across four ($L = 4$) layers;
- For each scenario: 150 synthetic multiplexes, community sizes vary between $[10, 100]$ nodes, within-community edge probability $0.25 \leq p \leq 0.40$.

Simulations: Comparative performance

Table: Performance of various algorithms in five simulated scenarios (NMI similarity)

	S1	S2	S3	S4	S5
LART	0.99 ± 0.02	0.89 ± 0.07	0.97 ± 0.03	0.98 ± 0.04	0.96 ± 0.06
MM	0.98 ± 0.04	0.81 ± 0.07	0.83 ± 0.04	0.97 ± 0.04	0.92 ± 0.09
IM	0.43 ± 0.07	0.64 ± 0.14	0.81 ± 0.11	0.60 ± 0.10	0.53 ± 0.09
PMM	0.95 ± 0.15	0.52 ± 0.16	0.68 ± 0.02	0.97 ± 0.07	0.84 ± 0.21
ST	0.69 ± 0.07	0.76 ± 0.13	0.83 ± 0.05	0.72 ± 0.04	0.71 ± 0.11
SM	0.68 ± 0.07	0.78 ± 0.12	0.84 ± 0.06	0.71 ± 0.05	0.72 ± 0.09

- For LART and MM: report best result over resolution parameter $\gamma = 0.25, 0.75, 1, 1.25, 1.50, 1.75, 2, 2.25, 2.5, 2.75, 3$.
- Consider $t = 3L$.
- $\varepsilon = 1$.

Simulations: Comparative performance

Table: Performance of competing algorithms in five simulated scenarios for different inter-layer weights (NMI similarity)

	S1	S2	S3	S4	S5
LART	0.99 ± 0.02	0.89 ± 0.07	0.97 ± 0.03	0.98 ± 0.04	0.96 ± 0.06
LART(ω=1)	0.96 ± 0.10	0.79 ± 0.12	0.97 ± 0.04	0.77 ± 0.05	0.90 ± 0.13
LART(ω=0.5)	0.84 ± 0.13	0.85 ± 0.12	0.93 ± 0.07	0.73 ± 0.02	0.87 ± 0.10
LART(ω=0.1)	0.69 ± 0.07	0.88 ± 0.08	0.81 ± 0.04	0.72 ± 0.04	0.73 ± 0.10
MM	0.98 ± 0.04	0.81 ± 0.07	0.83 ± 0.04	0.97 ± 0.04	0.92 ± 0.09
MM(ω=1)	1.00 ± 0.00	0.62 ± 0.13	0.67 ± 0.02	0.98 ± 0.03	0.88 ± 0.18
MM(ω=0.5)	0.84 ± 0.12	0.61 ± 0.14	0.82 ± 0.01	0.80 ± 0.04	0.79 ± 0.16
MM(ω=0.1)	0.73 ± 0.06	0.62 ± 0.13	0.82 ± 0.01	0.78 ± 0.05	0.72 ± 0.14
IM	0.43 ± 0.07	0.64 ± 0.14	0.81 ± 0.11	0.60 ± 0.10	0.53 ± 0.09
IM(ω=1)	0.43 ± 0.07	0.64 ± 0.14	0.81 ± 0.11	0.60 ± 0.10	0.53 ± 0.09
IM(ω=0.5)	0.89 ± 0.13	0.89 ± 0.05	0.80 ± 0.11	0.94 ± 0.07	0.81 ± 0.10
IM(ω=0.1)	0.89 ± 0.13	0.89 ± 0.05	0.80 ± 0.11	0.94 ± 0.07	0.81 ± 0.10
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- Adding white noise slightly decreases results but performance is barely affected by up to 10% of white noise edges.
- These results are valid for different number of layers $L = 2, 3, 4, 5$.

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- LART performs comparatively well to competing algorithms for detecting communities shared across all layers.
- LART is stable for different γ and t values.
- The introduced inter-layer weights and corresponding locally adaptive probabilities prove to be beneficial for shared and non-shared community detection.

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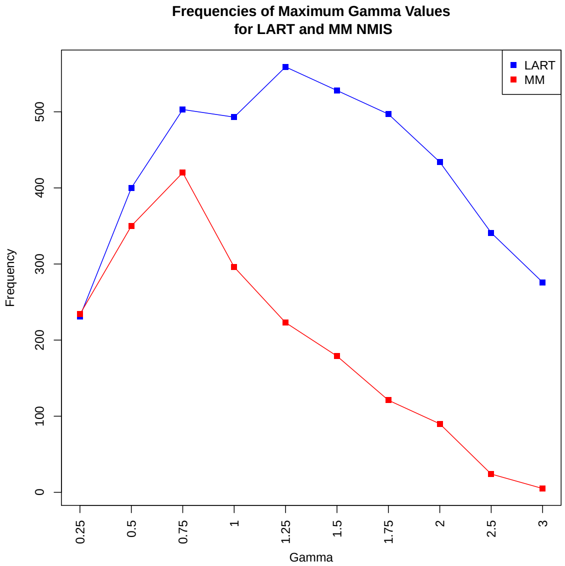
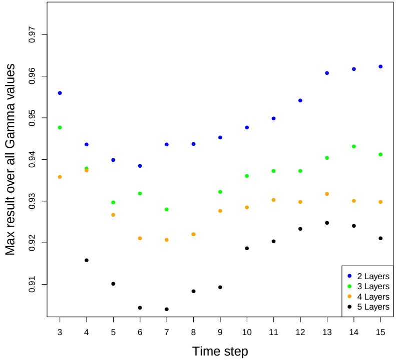
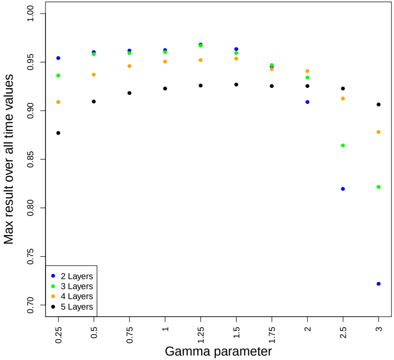
Simulations: Comparative performance



Simulations: Tuning Parameters

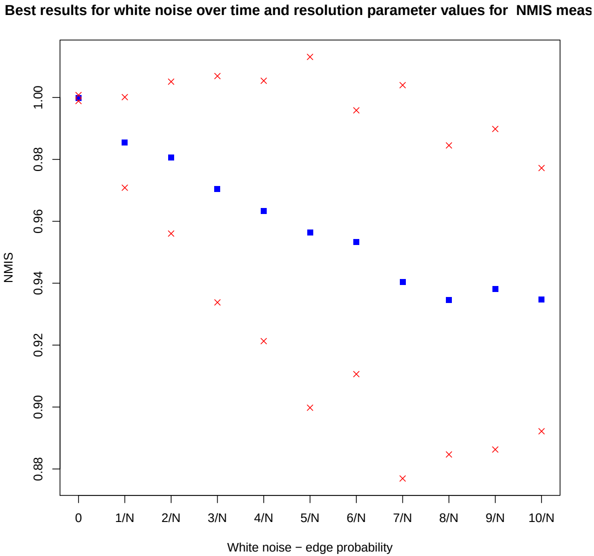


: Results for varying time steps t

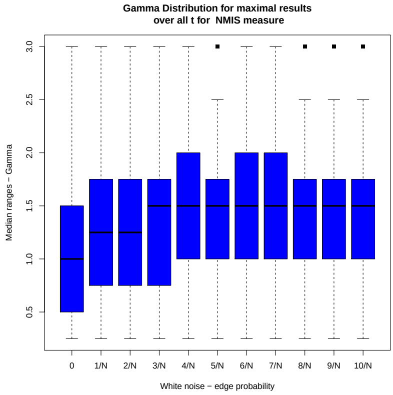


: Results for different γ parameters

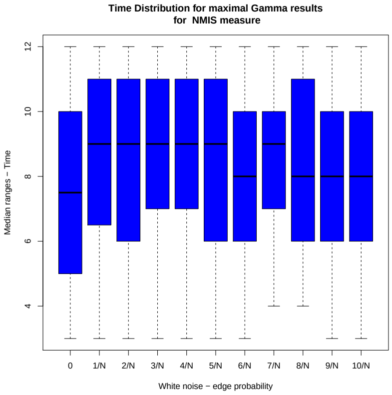
Simulations: White Noise



Simulations: White Noise



: Median ranges for Gamma values that produce best result at each white noise level.



: Median ranges for time values (over best Gamma result) that produce best result at each white noise level.