
Supervised rank aggregation approach for link prediction in complex networks

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ABSTRACT. In this paper we propose a new topological approach for link prediction in dynamic complex networks based on supervised rank aggregation. This functions as follows: first we rank a list of unlinked nodes in a network at instant t according to different topological measures, thus getting a ranked list for each measure. We assume that each topological measure should rank an actually occurring link on the top positions. Observing the network at instant $t + 1$ where some new links appear, we weight each topological measure according to its performances in predicting the new links. These learned weights are then used in a modified version of classical computational social choice algorithms (such as Borda, Kemeny, etc) in order to have a model for predicting new links. We show the effectiveness of this approach through different experimentation made on co-authorship networks extracted from the DBLP bibliographical database. Results we obtain are also compared with the outcome of classical supervised machine learning based link prediction approaches applied to the same data sets.

RÉSUMÉ. Dans ce travail, nous sommes intéressés à trouver une nouvelle approche de prévision de liens dans les grandes graphes basées sur la fusion de préférence. Nous ordonnons une liste de paires de noeuds non liés dans un réseau à l'instant t , selon différentes mesures topologiques, obtenant ainsi une liste ordonnée pour chaque mesure. Nous supposons que chaque mesure topologique doit classer un lien réel sur les postes de haut niveau. Observant le réseau à l'instant $t + 1$ où certains nouveaux liens apparaissent, nous donnons un poids à chaque mesure topologique en fonction de ses performances en prévision des nouveaux liens. Ces pondérations sont ensuite utilisées dans une version modifiée des algorithmes de vote (comme Borda, Kemeny, etc) pour avoir un modèle de prévision de nouveaux liens. Nous montrons l'efficacité de cette approche par l'expérimentation faite sur le réseaux bibliographiques extraites de la base de données DBLP.

KEYWORDS: Link prediction, Supervised rank aggregation, Social network analysis

MOTS-CLÉS : Prévision de liens, Fusion de préférence supervisée, Analyse de réseaux sociaux

1. Introduction

In analysis of real world complex networks, link prediction plays a vital role. It refers to the problem of finding a probable association between two unconnected nodes at a certain point of time by studying the history of evolution of network. It has a variety of application in many domains like recommender systems in collaborative networks, identification of probable professional or academic associations in scientific collaboration networks or e-commerce sites, identification of structures of criminal networks and structural analysis in the field of microbiology or bio-medicine etc. Many approaches have been proposed in the scientific literature. Recent surveys on the topic can be found in [LU 11, MOH 10].

A major trend is composed of *dyadic topological approaches*. In a seminal work proposed in [LIB 03], authors have shown that simple topological measures representing relationships between pairs of unlinked nodes in a complex network, can be used for predicting formation of new links. Let's consider the case of applying *common neighbors* as a topological measure. Let \mathcal{L} be the list of pairs of unlinked nodes (belonging to same connected component). We have $\mathcal{L} = \{(x, y)\}$. Let $\Gamma(x)$ be the function returning a set of direct neighbors of node x in the graph. The common neighbors function of two nodes x, y is then defined by:

$$CN(x, y) = |\Gamma(x) \cap \Gamma(y)| \quad (1)$$

The list \mathcal{L} is sorted according to the values obtained by applying the common neighbors function to couples of unlinked nodes. The top k couples of nodes are then returned as the output of the prediction task. The assumption here is that, the more a couple of unlinked nodes share common neighbors, the more they are likely to have a link in future. In [LIB 03] k is equal to the number of really appearing links. Other types of topological measures can be applied for the same purpose.

Many other works have been published focusing on how to combine different topological metrics in order to enhance prediction performances. One widely applied approach is based on expressing the problem of link prediction as a problem of binary classification. The idea is to compute for each unlinked couple of nodes in \mathcal{L} , a set of topological measures. Then with each element in \mathcal{L} , associate one of the labels: *Linking* or *Not-linking* based on the status of the graph at a future step. The dataset hence computed (topological features with classes) can then be used to learn a model, for discriminating the *linking* class from the *not-linking* one using classical supervised machine learning approaches [HAS 06, BEN 10].

Surprisingly, none of the previous works, attempted to combine the prediction power of individual topological measures by applying computational social choice algorithms (or what is also known as rank aggregation methods) [CHE 07]. *Rank aggregation* can be defined as a process of combining a number of ranked lists or rankings of candidates or elements to get a single list and with least possible disagreement with all the experts or voters who provide these lists. These methods were a part of social choice theory and were mostly applied to political and election related prob-

lems [BOR 81, BLA , YOU 78]. These techniques were designed to ensure fairness amongst experts while combining their rankings and hence all experts are given equal weights. Expressing the link prediction problem in terms of a vote is straightforward: candidates are examples (pairs of unconnected nodes), while voters are topological measures computed for these pairs of unlinked nodes. Then we have a voting problem with quite huge set of candidates and rather a reduced set of voters. These settings are very similar to those encountered when considering the problem of ranking documents in a meta-search engines where voting schemes has also been applied with success [DWO 01b, ASL 01, MON 02].

In our settings, prediction performances can be boosted by weighting differently the different applied topological measures (voters) in function of their individual performances in predicting new links. We propose here two different weighting scheme. Weights are used in two different weighted rank aggregation methods: The first one is based on the classical Borda count approach [BOR 81], while the second is based on the Kemeny aggregation rule. The later is known to compute the *Condorcet* winner of an election (if it exists): the candidate that wins each duel with all other candidates. We contribute in three ways: first we provide a way to generate weights for the topological measures; second, we propose a new way of introducing weights to approximate Kemeny aggregation; and third, we use supervised or weighted rank aggregation to link prediction task in complex networks. Our approach is evaluated in the context of a link prediction task applied to academic co-authorship networks. Experiments are conducted on real networks extracted from the now well known DBLP bibliographical server.

The reminder of this paper is organized as follows. Next in section 2 we describe briefly some related work applying rank aggregation in similar settings. Our approach is detailed in section 3. Experiments are presented in section 4. Results are provided and commented in the same section. A conclusion is given in section 5.

2. Related Work

In the work of [HAS 06], authors propose to use supervised machine learning algorithms for link prediction. They convert the problem of link prediction in graphs into a binary classification problem where examples are unconnected node pairs and are characterized by a vector of topological attribute values. They make a comparative analysis on the suitability of many learning algorithms to be used in link prediction based on their prediction performance. Another interesting study that they made was to rank the attributes based on various factors in order to compare and judge their relative strength in a prediction task.

The work presented in [BEN 10] is a temporal approach for link prediction based on supervised machine learning where link prediction is done by using Decision tree algorithm with boosting. This is a dynamic approach where the evolution of the network is taken into account. Their work is mostly based on bipartite graphs and they

introduce the concept of indirect topological measures computed using the projected graphs. They show how the use of indirect attributes greatly enhance the final prediction result.

Looking into the work based on rank aggregation techniques, we can say that not much have been explored when it comes to application of rank aggregation in link prediction. Moreover these work apply mostly unsupervised rank aggregation algorithms giving equal weight to all the experts who provide the ranked lists.

One of the well known work is weighted majority algorithm proposed in [LIT 89] where the authors have proposed to use weights for predictors, all having equal weights in the beginning. There is a master predictor which makes the final prediction based on the class which corresponds to a maximum total weights of predictors. If the final prediction is wrong then weights of all predictors who disagreed with that label, is increased by a factor β such that $0 \leq \beta < 1$ and thus reducing the effect of unworthy predictors at each iteration. This approach has a limitation that the performance of the master predictor can be at most equal to the best performing predictor. On the contrary, the use of rank aggregation can provide even better prediction at times. This may be due the fact that, in these algorithms, the “likes” of majority of the predictors is given higher preference. At the same time, the “dislikes” are given least preference. So these algorithms are much more spam/noise resistant.

A significant work on supervised rank aggregation has been done in [LIU 07] where authors propose supervised aggregation by Markov chain to enhance the ranking result on meta-searches. However, it has been shown that Local Kemenization improves on Markov chain-based approaches [DWO 01a].

Another very recent work is in [SUB 11] where the authors use supervised rank aggregation to find influential nodes and future links. Authors have proposed their own supervised Kemeny aggregation method based on quick sort and applied it to Twitter and citation networks. However, their method is mostly based on the topological features of nodes. Where as our work is based on the features of a couple of nodes(edges) with a use of merge sort algorithm to find supervised local Kemeny aggregation. The reason why we use merge sort is that it is seemingly more stable than quick sort. Stability of a sorting algorithm is important when two candidates have equal importance. A stable sorting algorithm is the one which never affects the relative order of two candidates who are equal in ranks. This feature may be important when we have ties.

3. Link Prediction by Supervised Rank Aggregation

Let $G = \langle G_1, \dots, G_n \rangle$ be a temporal sequence of an evolving graph. The whole sequence is divided into three parts: *training*, *labeling* and *testing* or *validation*. Three graphs namely G_{learn} , G_{label} and G_{test} are generated by making union of the temporal sequences of the graphs for three corresponding time slots. The training data is constructed as follows. An example will be generated for each couple of nodes

(x, y) that are not linked in G_{learn} but both belonging to the same connected component. The class labeling is obtained by checking whether the couple of nodes is indeed connected in G_{label} . If such a connection exists then it will be a positive example in the supervised learning task and if no connection exists, it will be a negative example [BEN 10]. Thus, examples are generated from these graphs for both training and validation. These examples are also characterized by a given number of topological attributes.

Each attribute of an example has the capacity to provide some unique information about the data when considered individually. The training examples are ranked based on the attribute values. So, for each attribute we will get a ranked list of all examples. Considering only the top k ranked examples and with an assumption that when we rank the examples according to their attribute values, the positive examples should be ranked on the top, we compute the performance of each attribute. This performance is measured in terms of either *precision* (maximization of identification of positive examples) or *false positive rate* (minimization of identification of negative examples) or a combination of both. Based on the individual performances, a weight is assigned to each attribute.

For validation, we use examples obtained from the validation graph characterized by same attributes and try to rank all examples based on their attribute values. So for n different attributes we shall have n different rankings of the test examples. These ranked lists are then merged using a *supervised rank aggregation* method and the *weights of the attributes* obtained during learning process. The top k ranked examples in the aggregation are taken to be the predicted list of positive examples. Using this predicted list, we calculate the performance of our approach. k in this case is equal to the number of positive examples in the validation graph.

3.1. Weights computation

We propose to compute voters (topological measures) weights based on their capability to identify correct elements in top k positions of their rankings. Weights associated to applied topological measures are computed based on the following criteria :

– **Maximization of positive precision:** Based on maximization of identification of positive examples the attribute weight is calculated as

$$w_i = n * Precision_i \quad (2)$$

where n is the total number of attributes and $Precision_i$ is the *precision* of attribute i based on identification of positive examples. Just to remind, precision is defined as the fraction of retrieved instances that are relevant.

– **Minimization of false positive rate:** By minimizing the identification of negative examples we get a weight as below

$$w_i = n * (1 - FPR_i) \quad (3)$$

where n is the total number of attributes and FPR_{a_i} is the *false positive rate* of attribute a_i based on identification of negative examples. False positive rate is defined as the fraction of non-relevant instances that are retrieved as relevant.

3.2. Supervised rank aggregation

First let's define some basic functions used later in defining weighted aggregation functions. Let L_i be a ranked list of n candidates (a vote). $L_i(x)$ denotes the rank of element x in the list L_i . The top ranked element has the rank 0. The basic individual Borda score of an element x for a voter i is then given by:

$$B_i(x) = n - L_i(x)$$

Let x and y be two candidates. We define the local preference function as follows :

$$Pref_i(x, y) = \begin{cases} 1 & \text{if } B_i(x) > B_i(y) \\ 0 & \text{if } B_i(x) < B_i(y) \end{cases} \quad (4)$$

Introducing weights in Borda aggregation rule is rather straightforward: Let (w_1, w_2, \dots, w_r) be the weights for r voters providing r ranked lists on n candidates. The weighted Borda score for a candidate x is then given by:

$$B(x) = \sum_{i=1}^r w_i * B_i(x) \quad (5)$$

For approximate Kemeny aggregation[DWO 01a] we introduce weights into the definition of the non-transitive preference relationships between candidates. This is modified as follows. Let w_T be the sum of all computed weights i.e. $w_T = \sum_{i=1}^r w_i$. For each couple of candidates x, y we compute a score function as follows:

$$score(x, y) = \sum_{i=1}^r w_i * Pref_i(x, y)$$

The weighted preference relation(\succ_w) is then defined as follows :

$$x \succ_w y : score(x, y) > \frac{w_T}{2}$$

This new preference relation is used to sort an initial aggregation of candidates in order to obtain a supervised Kemeny aggregation. The initial aggregation can be any of the input lists or an aggregation obtained by applying any other classical aggregation method like Borda. In our algorithm, we have applied merge-sort for the time being.

4. Experiments

We evaluated our approach using data obtained from DBLP¹ databases. DBLP is a scientific bibliography website containing a large database of articles mostly related

1. <http://www.dblp.org>

to computer science. Our network consists of authors and their publications providing us a bipartite structure of graph. The data used corresponds to a time span of 1970 to 1976. This data is divided into two datasets containing information for different years, each having a training set and a test or validation set. We generate examples for each dataset. Each example is a pair of unconnected nodes, represented by a vector of topological measures and a class label. Tables.1 and 2 provide information about the training graphs while table.3 summarizes information about the examples generated.

Datasets	Training Time	Graph		
		Authors	Publications	Edges
Dataset1	[1970,1973]	2661	1487	6634
Dataset2	[1972,1975]	4536	2542	10855

Table 1. Author-Publication bipartite graphs

Datasets	Author Graph		Publication Graph	
	Nodes	Edges	Nodes	Edges
Dataset1	2661	2575	1487	1520
Dataset2	4536	4510	2542	2813

Table 2. Projected Graphs

Datasets	Training Time	Labeling Time	Testing Time	Training examples		Test examples	
				Pos	Neg	Pos	Neg
Dataset1	[1970,1973]	[1974,1975]	[1971,1974]	30	1663	41	3430
Dataset2	[1972,1975]	[1976,1977]	[1973,1976]	87	19245	82	18675

Table 3. Examples generated from Author graphs

The attributes characterizing each example in each dataset are:

– **Neighborhood-based attributes:**

- Common neighbors : $VC(x, y) = \|\Gamma(x) \cap \Gamma(y)\|$
- Jaccard's coefficient : $JC(x, y) = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{\|\Gamma(x) \cup \Gamma(y)\|}$
- Adamic Adar: $AD(x, y) = \sum_{z \in \Gamma(x) \cap \Gamma(y)} \frac{1}{\log \|\Gamma(z)\|}$ [ADA 03]
- Preferential attachment: $AP(x, y) = \|\Gamma(x) \times \Gamma(y)\|$ [HUA 05]

– **Distance-based attributes:**

- Shortest path distance (*Dis*)

- Katz: $Katz(x, y) = \sum_{\ell=1}^{\infty} \beta^{\ell} \times \| path_{x,y}^{(\ell)} \|$, where $path_{x,y}^{(\ell)}$ is the number of paths between x and y of length ℓ and β is a positive parameter which favours shortest paths [KAT 53]

- Maximum forest algorithm (*MFA*): It makes use of *Laplacian* matrix and identity matrices of the graph to find distance between all the nodes of a graph. A matrix is obtained as $M = (I + M_L)^{-1}$ where I is an identity matrix and M_L is the *Laplacian* of the graph being studied. So, $MFA(x, y) = M(x, y)$ [FOU 07].

– **Centrality-based attributes:**

- Product of PageRank (*PPR*)[BRI 98]
- Product of clustering coefficient (*PCF*)
- Product of degree centrality (*PCD*)

These attributes can be computed directly from the bipartite graph and/or also from the projected graphs. Projected graph refers to the unimodal graphs obtained by projecting the bipartite graph over one of its node sets [BEN 10]. The attributes computed from projected graphs are called indirect attributes. Before using the rank aggregation method, we ranked all the test examples by their attribute values. Considering only the *top - k* examples and taking the number of positive examples in each dataset as the corresponding value of k , we compute the performance of each attribute in identifying the real positive links in *top - k* positions. Table.4 summarizes this information. In the first part of experimentation we applied our approach to the complete datasets. For rank aggregation, we have used supervised Borda and supervised Kemeny methods. We also try to compare our approach with link prediction approaches using basic machine learning algorithms like Decision tree, Naive bayes etc. We name our approaches as Supervised Borda 1 and Supervised Borda 2 based on how the attribute weights are computed. 1 represents weights computed based on maximization of positive precision and 2 represents weights being computed based on minimization of false positive rates. We will follow the same convention to represent supervised Kemeny. Figure.1 shows the results obtained on the complete datasets in terms of precision. For both datasets, our approach of supervised Kemeny based on maximization of positive precision, gives a comparatively better performance in terms of precision as compared to most of the other methods.

In the second part of experimentation we try to evaluate the performance of our approach for the task of recommendation. So we vary the value of k used during validation to observe the performance on dataset-1. We vary k from 5 to total number of positive examples in the test set of dataset-1 i.e. 41. Figure.2 show the performances of our rank aggregation based approaches in terms of precision and recall. As observed from figures, the precision decreases while the recall increases with increasing k . The performance of supervised Kemeny is much better than that of supervised Borda, which is quite evident due to the fact that Kemeny based approaches are quite efficient in dealing with noise or spam in data. The precision of supervised and approximate Kemeny approaches at lower values of k are good enough to make them suitable for recommendation task.

Attributes	Dataset1	Dataset2
Katz	0	0
MFA	0.0244	0.0732
PPR	0.0244	0.0244
PCF	0.0732	0.0244
PCD	0	0
VC	0.5122	0.4268
JC	0.2195	0.1707
AD	0.1463	0.1463
AP	0.0488	0
Dis	0	0.0122
Indirect Katz	0.1220	0.1098
Indirect MFA	0.0488	0.0732
Indirect PPR	0.0488	0
Indirect PCF	0.4878	0.4756
Indirect PCD	0.0244	0.0122
Indirect VC	0.0488	0.0488
Indirect JC	0.0488	0.1098
Indirect AD	0.0976	0.0488
Indirect AP	0.0244	0
Indirect Dis	0.6098	0.5366

Table 4. Results (precision) obtained by ranking the test examples by attribute values

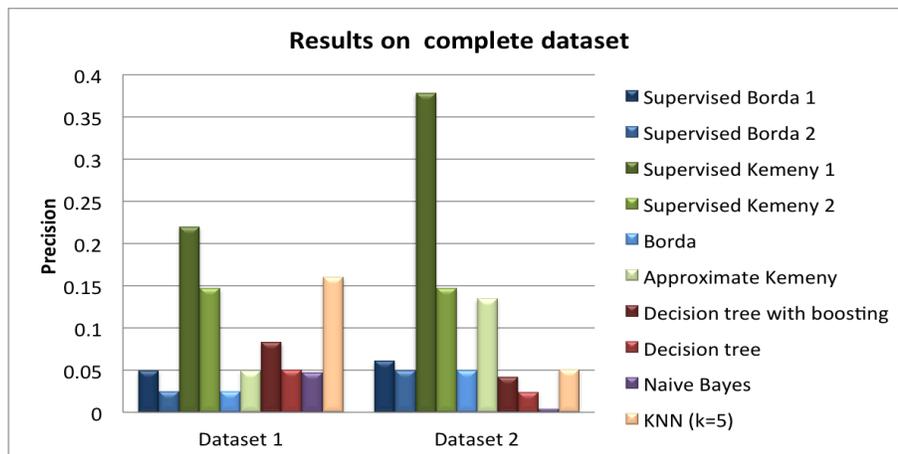


Figure 1. Precision for complete test set by learning on complete datasets

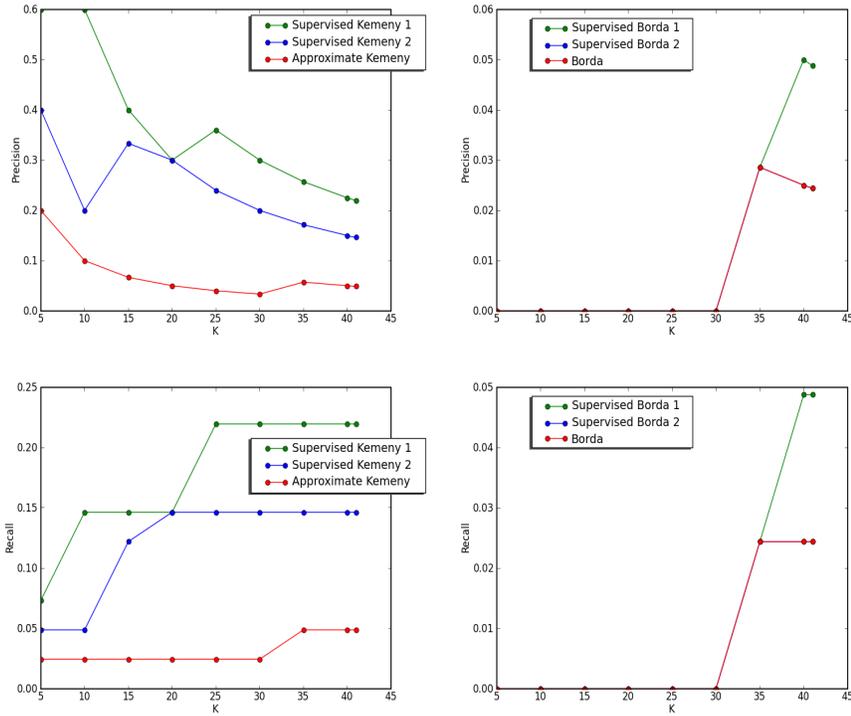


Figure 2. Effect on performance by varying k on Dataset-1

5. Conclusion

This paper presents a new solution to link prediction problem in dynamic large graphs representing complex networks. We propose a novel approach based on supervised rank aggregation and is motivated by the belief that each attribute can provide us with some unique information which can be aggregated in the end to make a better prediction of association between two unconnected entities in a network. First we have come up with a new way of introducing weights in a well known rank aggregation method. And secondly, we have proposed to apply this approach for the purpose of link prediction in complex networks. We evaluated our approach on a co-authorship network obtained from DBLP database. The experimental results are quite encouraging as our method seems to perform better than the approaches using classical machine learning algorithms for link prediction. We also tested performance of the approach by varying the value of k for validation. This experiment was done in order to see if the method can perform well with very small values of k which a primary requirement for recommendation task. In future we intend to apply our method on other scientific collaboration networks to further verify the performance of our approach. While

working in complex networks, it is extremely important to take care of the complexity of the algorithms one is working on. In our case we have made a big attempt of trying to apply rank aggregation algorithms which have their own limitations in terms of time complexity. In order to avoid the limitations of Kemeny optimal aggregation (which is NP-hard), we have focused on the concept of approximate Kemeny aggregation. And the use of merge sort have reduced the complexity to $O(rn \log n)$ where r is the number of experts and n is the number of candidates in each input ranked list. In order to enhance the performance of our approach and reduce the complexity further, we are looking forward to use the concept of $top - k$ aggregation, where instead of aggregating the complete input lists, only the $top - k$ from each are considered for aggregation.

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